Midterm 1

EECS 245, Spring 2026 at the University of Michigan

Name: ______________________________

uniqname: ______________________________

UMID: ______________________________

Room: ◯ 1690 BBB (1-3PM) ◯ 1690 BBB (extended time)

Instructions

- This exam consists of 9 problems, worth 100 points, spread across 12 pages (6 sheets of paper).
- You have 120 minutes to complete this exam, unless you have extended-time accommodations through SSD.
- Write your uniqname in the top right corner of each page.
- For free response problems, **you must show all of your work**, and [circle] your final answer. We will not grade work that appears elsewhere, and you may lose points if your work is not shown.
- For multiple choice problems, completely fill in bubbles and square boxes; if we cannot tell which option(s) you selected, you may lose points.
  ◯ A bubble means that you should only select one choice.
  ☐ A square box means that you should select all that apply.
- You may refer to **one double-sided 8.5x11" handwritten notes sheet**. Other than that, you may not refer to any other resources or technology during the exam (no phones, watches, or calculators).

You are to abide by the University of Michigan/Engineering Honor Code. To receive a grade, please sign below to signify that you have kept the Honor Code pledge.

I have neither given nor received aid on this exam, nor have I concealed any violations of the Honor Code.

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Tip: Skim through the entire exam before starting to work on it.

Problem 1 (16 pts)

Suppose we'd like to find the optimal parameter, w^*, for the constant model $h(x_i) = w$, using the following dataset of $n = 4$ values, y_1, y_2, y_3, y_4:

$$0, \quad 2, \quad 4, \quad 20$$

a) (3 pts) First, suppose we find the optimal parameter by minimizing mean squared error, $R_{\text{sq}}(w)$. Which value of w minimizes $R_{\text{sq}}(w)$? Give your answer as a number with no variables.

minimizer of $R_{\text{sq}}(w) =$ ☐

Now, consider the **clipped** loss function, defined below.

$$L_{\text{clip}}(y_i, h(x_i)) = \min\{(y_i - h(x_i))^2, 9\}$$

For example, $L_{\text{clip}}(10, 5) = 9$ and $L_{\text{clip}}(5, 3) = 4$.

Let $R_{\text{clip}}(w)$ be the average clipped loss for the constant model and this dataset.

b) (3 pts) State one value of w where the derivative of $R_{\text{clip}}(w)$ is not defined.

one value of w where the derivative of $R_{\text{clip}}(w)$ is not defined = ☐

c) (3 pts) Suppose we restrict w to the interval $1 \leq w \leq 3$. Among all values of w in this interval, which value minimizes $R_{\text{clip}}(w)$? Give your answer as a number with no variables.

minimizer of $R_{\text{clip}}(w)$ within the interval $[1, 3] =$ ☐

d) (3 pts) Now suppose there are no restrictions on w. Among all possible values of w, which value minimizes $R_{\text{clip}}(w)$? Give your answer as a number with no variables.

minimizer of $R_{\text{clip}}(w) =$ ☐

e) (4 pts) State one pro and one con of using clipped loss instead of squared loss to find optimal model parameters.

☐

Problem 2 (10 pts)

We will continue to use the constant model, $h(x_i) = w$, and the same dataset of $n = 4$ values as in Problem 1:

$$0, \quad 2, \quad 4, \quad 20$$

Instead of the clipped loss function, consider the **weighted absolute** loss function, defined below.

$$L_{\text{WA}}(y_i, h(x_i)) = \begin{cases} \beta(y_i - h(x_i)), & h(x_i) < y_i \\ h(x_i) - y_i, & h(x_i) \geq y_i \end{cases}$$

where β is a positive integer. Let $R_{\text{WA}}(w)$ be the average weighted absolute loss for the constant model and this dataset.

The slope of $R_{\text{WA}}(w)$ at w, for any value of w not equal to one of the y_i values, is

$$\text{slope of } R_{\text{WA}}(w) \text{ at } w = \frac{\text{\# left of } w - \beta(\text{\# right of } w)}{4}$$

a) (4 pts) Suppose $\beta = 1$. Which value of w minimizes $R_{\text{WA}}(w)$? Show your work, and write your final answer in the box provided. Your answer should be a number with no variables. If there are multiple possible answers, state just one.

minimizer of $R_{\text{WA}}(w) =$

b) (6 pts) Now suppose $\beta = 2$. Which value of w minimizes $R_{\text{WA}}(w)$? Show your work, and write your final answer in the box provided. Your answer should be a number with no variables. If there are multiple possible answers, state just one.

minimizer of $R_{\text{WA}}(w) =$

Problem 3 (14 pts)

Suppose we fit a simple linear regression model **with** an intercept term, $h(x_i) = w_0 + w_1 x_i$, to a dataset of n points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ by minimizing mean squared error. You are given the following information:

- The fit model satisfies $h(-4) = 5$ and $h(8) = 14$.
- The mean of $y_1, y_2, \ldots, y_n$ is $\bar{y} = 2$.

a) (6 pts) Find $\bar{x}$, the mean of $x_1, x_2, \ldots, x_n$. Show your work, and write your final answer in the box provided. Your answer should be a number with no variables. *Hint: What property does the line $h(x_i)$ satisfy?*

$\bar{x} =$ ______

b) (4 pts) Suppose the correlation coefficient between the x-values and y-values is $r = 1/3$. The standard deviation of y, σ_y, is c times the standard deviation of x, σ_x. In other words,

$$\sigma_y = c\sigma_x$$

What is the value of c?

◯ $1/4$ ◯ $4/9$ ◯ $3/4$ ◯ $9/4$ ◯ 3 ◯ 4

c) (4 pts) Let $e_i = y_i - h(x_i)$ be the fit model's error for the ith point. Note that e_i may either be positive or negative. Which of the following statements are **guaranteed** to be true? **Select all** that apply.

☐ $\sum_{i=1}^{n} e_i = 0$ ☐ $\sum_{i=1}^{n} x_i e_i = 0$ ☐ $\sum_{i=1}^{n} y_i e_i = 0$ ☐ $\sum_{i=1}^{n} e_i(x_i - \bar{x}) = 0$

Problem 4 (8 pts)

Let $\vec{u}, \vec{v} \in \mathbb{R}^n$ be vectors satisfying

$$\|\vec{v}\| = 5, \qquad \|\vec{u} + \vec{v}\| = 10, \qquad \|\vec{u} - \vec{v}\| = 6$$

Find $\|\vec{u}\|^2$ (**not** $\|\vec{u}\|$). Show your work, and write your final answer in the box provided. Your answer should be a number with no variables.

$\|\vec{u}\|^2 =$ ☐

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Problem 5 (13 pts)

Suppose $\vec{u}, \vec{v} \in \mathbb{R}^n$ are non-zero vectors and k is a scalar. Let

$$f(k) = \|\vec{u} - k\vec{v}\|^2 + Ck^2$$

where $C \geq 0$ is a non-negative constant.

a) (6 pts) In this part only, suppose $C = 0$, $\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. Find the value of k that minimizes $f(k)$. Show your work, and write your final answer in the box provided. Your answer should be a number with no variables.

minimizer of $f(k) =$ ☐

b) (4 pts) Note that $f(k)$ *almost* looks like the squared norm of the vector $\vec{u} - k\vec{v}$, but with an extra term Ck^2. Let's try and rewrite $f(k)$ so that it *is* the squared norm of another related vector. Define two new vectors, $\vec{U}, \vec{V} \in \mathbb{R}^{n+1}$ by appending the scalar a to the end of $\vec{u}$ and the scalar b to the end of $\vec{v}$.

$$\vec{U} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \\ a \end{bmatrix}, \quad \vec{V} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \\ b \end{bmatrix}$$

Select values of a and b so that $f(k) = \|\vec{U} - k\vec{V}\|^2$, for all possible non-negative values of C.

(i) What is the value of a? ◯ 0 ◯ C ◯ C^2 ◯ $\sqrt{C}$

(ii) What is the value of b? ◯ 0 ◯ C ◯ C^2 ◯ $\sqrt{C}$

c) (3 pts) As C increases, what happens to the value of k that minimizes $f(k)$? Explain your reasoning.

Problem 6 (11 pts)

Suppose $c \in \mathbb{R}$ is a constant and

$$\vec{u} = \begin{bmatrix} 3 \\ 1 \\ c \end{bmatrix}, \qquad \vec{v} = \begin{bmatrix} 6 \\ c \\ -2 \end{bmatrix}$$

a) (4 pts) Fill in the blanks to complete the sentence:

For all values of c, span($\{\vec{u}, \vec{v}\}$) is a __(i)__-dimensional subspace of __(ii)__.

(i): [ ] (ii): [ ]

b) (7 pts) Suppose the plane spanned by $\vec{u}$ and $\vec{v}$ is

$$ax + 24y + 3z = 0$$

where a is also a constant. Find the value of c. Show your work in the space provided, and write your final answer in the box provided. Your answer should be a number with no variables.

$c =$ [ ]

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Problem 7 (10 pts)

Suppose $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4 \in \mathbb{R}^n$ are a **linearly independent** collection of vectors. Define

$$\vec{p} = \vec{v}_1 + \vec{v}_2, \qquad \vec{q} = \vec{v}_2 + \vec{v}_3, \qquad \vec{r} = \vec{v}_3 + \vec{v}_4, \qquad \vec{s} = \vec{v}_4 + \vec{v}_1$$

a) (7 pts) Are $\{\vec{p}, \vec{q}, \vec{r}, \vec{s}\}$ linearly independent?

(i) Select an answer: ◯ Yes ◯ No

(ii) Prove your answer using the formal definition of linear independence. *Hint: You did something similar in Homework 4, Problem 6.*

b) (3 pts) What is the dimension of span($\{\vec{p}, \vec{q}, \vec{r}, \vec{s}\}$)? Give your answer as a number with no variables.

$\dim(\text{span}(\{\vec{p}, \vec{q}, \vec{r}, \vec{s}\})) =$ ☐

Problem 8 (8 pts)

Suppose S is the subspace of $\mathbb{R}^4$ defined by

$$S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \mathbb{R}^4 : x_1 - x_2 + x_3 - x_4 = 0 \right\}$$

Which of the following sets is a basis for S? **Select all** that apply.

☐ $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

☐ $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

☐ $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

☐ $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

Problem 9 (10 pts)

a) (7 pts) Suppose x and y are non-negative numbers. Using the Cauchy-Schwarz inequality, prove that

$$\frac{(x+y)^2}{2} \le x^2 + y^2$$

Solutions that do not use the Cauchy-Schwarz inequality will not receive credit.

b) (3 pts) Now suppose x, y, and z are non-negative numbers. Which inequality is guaranteed to be true?

- ◯ $\frac{(x+y+z)^2}{2} \le x^2 + y^2 + z^2$
- ◯ $\frac{(x+y+z)^2}{3} \le x^2 + y^2 + z^2$
- ◯ $\frac{(x+y+z)^2}{2} \le x^3 + y^3 + z^3$
- ◯ $\frac{(x+y+z)^3}{3} \le x^3 + y^3 + z^3$
- ◯ None of the above

Congrats on finishing Midterm 1! Feel free to draw us a picture about EECS 245 in the box below.