Midterm 1

EECS 245, Winter 2026 at the University of Michigan

Name: ______________________________

uniqname: ______________________________

UMID: ______________________________

Room: ◯ 1670 BBB (big room) ◯ 1690 BBB ◯ Other

Instructions

- This exam consists of 7 problems, worth a total of 100 points, spread across 12 pages (6 sheets of paper).
- You have 120 minutes to complete this exam, unless you have extended-time accommodations through SSD.
- Write your uniqname in the top right corner of each page.
- For free response problems, **you must show all of your work**, and circle your final answer. We will not grade work that appears elsewhere, and you may lose points if your work is not shown.
- For multiple choice problems, completely fill in bubbles and square boxes; if we cannot tell which option(s) you selected, you may lose points.
  ◯ A bubble means that you should only select one choice.
  ☐ A square box means you should select all that apply.
- You may refer to **one double-sided 8.5x11" handwritten notes sheet**. Other than that, you may not refer to any other resources or technology during the exam (no phones, watches, or calculators).

You are to abide by the University of Michigan/Engineering Honor Code. To receive a grade, please sign below to signify that you have kept the Honor Code pledge.

I have neither given nor received aid on this exam, nor have I concealed any violations of the Honor Code.

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Tip: Skim through the entire exam before starting to work on it.

Problem 1 (16 pts)

Consider a dataset of n values, $y_1, y_2, \ldots, y_n$, with:

- a mean of $\bar{y} = 18$
- a median of 15
- a standard deviation of $\sigma_y = 7$

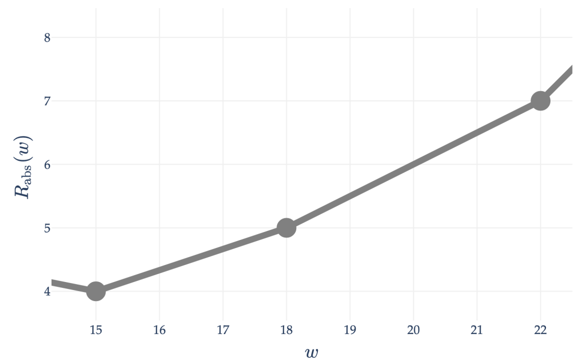
a) (4 pts) In the space provided, sketch the graph of $R_{\text{sq}}(w)$, the mean squared error of a constant prediction w on the dataset. For full credit:

- The shape of the graph must be correct.
- You must clearly label the coordinates of the **minimum point** on the graph.

b) (6 pts) Which of the following quantities is **guaranteed** to be equal to 0? Select all that apply.

- ☐ $\frac{1}{n}\sum_{i=1}^{n}(y_i - 15)$
- ☐ $\frac{1}{n}\sum_{i=1}^{n}(y_i - 18)$
- ☐ $\frac{1}{n}\sum_{i=1}^{n}(y_i - 15)^2$
- ☐ $\frac{1}{n}\sum_{i=1}^{n}(y_i - 18)^2$
- ☐ $\frac{1}{n}\sum_{i=1}^{n}(y_i - 15)^2 - 7^2$
- ☐ $\frac{1}{n}\sum_{i=1}^{n}(y_i - 18)^2 - 7^2$

c) (6 pts) Recall that $R_{\text{abs}}(w)$ is the mean absolute error of a constant prediction w on the dataset. A snippet of the graph of $R_{\text{abs}}(w)$ is shown below.



For clarity, the circles at $(15, 4)$, $(18, 5)$, and $(22, 7)$ indicate the points at which the slope of $R_{\text{abs}}(w)$ changes.

Given that there are $n = 72$ values in the dataset, how many values in the dataset are equal to **18**? Show your work and circle your final answer, which should be an integer with no variables.

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Problem 2 (14 pts)

Suppose we'd like to fit a simple linear regression model to a dataset of n points, $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, by minimizing mean squared error.

Suppose w_0^* and w_1^* are the optimal intercept and slope parameters, respectively, and let

$$M = \frac{1}{n} \sum_{i=1}^{n} (y_i - (w_0^* + w_1^* x_i))^2$$

Finally, let σ_x and σ_y be the standard deviations of the x-values and y-values in the dataset, respectively. Assume that $\sigma_x > 0$ and $\sigma_y > 0$.

a) (5 pts) Which of the following is the relationship between M and σ_y^2? Select an answer and provide a brief explanation in the box provided.

◯ $M \leq \sigma_y^2$ ◯ $M = \sigma_y^2$ ◯ $M \geq \sigma_y^2$ ◯ Impossible to tell

b) (5 pts) Suppose that $M = 0$. What is the value of r, the correlation coefficient between the x-values and y-values in the dataset? Circle your final answer and provide a brief explanation. If there are multiple possible values, state them all.

c) (2 pts) True or False: It is possible for there to be multiple pairs of $(\text{intercept}, \text{slope})$ with a mean squared error of M.

◯ True ◯ False

d) (2 pts) True or False: It is possible for there to be multiple pairs of $(\text{intercept}, \text{slope})$ with a mean squared error of $M + 1$.

◯ True ◯ False

Problem 3 (12 pts)

Consider the following two planes, P_1 and P_2, in $\mathbb{R}^3$.

- P_1 is the plane spanned by the vectors $\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ -4 \\ -3 \end{bmatrix}$.
- P_2 is the plane defined by the equation $5x + 3y - z = 0$.

a) (6 pts) Find the equation of P_1 in standard form, i.e. $ax + by + cz + d = 0$. Show your work and circle your final answer.

b) (6 pts) Planes P_1 and P_2 intersect at a line. Find the equation of this line in parametric form. Show your work and circle your final answer. *Hint: This can be done without knowing the answer to the previous part.*

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Problem 4 (12 pts)

Suppose $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\vec{u} + \vec{v} + \vec{w} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$. Assume that none of $\vec{u}$, $\vec{v}$, or $\vec{w}$ are the zero vector, $\vec{0}$.

For each statement below, identify whether it is **impossible**, **possible**, or **guaranteed**, and provide a brief explanation in the box provided.

a) (4 pts) $\vec{u}$ and $\vec{v}$ are orthogonal.

◯ Impossible ◯ Possible ◯ Guaranteed

b) (4 pts) The set $\{\vec{u}, \vec{v}, \vec{w}\}$ is linearly dependent.

◯ Impossible ◯ Possible ◯ Guaranteed

c) (4 pts) $\vec{u}$, $\vec{v}$, and $\vec{w}$ all have the same norm (length).

◯ Impossible ◯ Possible ◯ Guaranteed

Problem 5 (12 pts)

Suppose $\vec{u}, \vec{v} \in \mathbb{R}^n$. Let $\vec{p}$ be the projection of $\vec{u}$ onto $\vec{v}$. Furthermore, we know that:

$$\underbrace{\|\vec{v}\| = 2}_{\text{length of } \vec{v} \text{ (not } \vec{u})} \qquad \|\vec{p}\| = 3$$

a) (6 pts) Find $|\vec{u} \cdot \vec{v}|$. Show your work and circle your final answer, which should be a number with no variables.

b) (6 pts) For each pair of vectors, determine whether they are orthogonal, linearly dependent, or neither. Make sure to select **one bubble per row**.

	pair of vectors	orthogonal	linearly dependent	neither
(i)	$\vec{u}$ and $\vec{u} - \vec{p}$	◯	◯	◯
(ii)	$\vec{u}$ and $\vec{v} - \vec{p}$	◯	◯	◯
(iii)	$\vec{v}$ and $\vec{u} - \vec{p}$	◯	◯	◯
(iv)	$\vec{v}$ and $\vec{v} - \vec{p}$	◯	◯	◯
(v)	$\vec{p}$ and $\vec{u} - \vec{p}$	◯	◯	◯
(vi)	$\vec{p}$ and $\vec{v} - \vec{p}$	◯	◯	◯

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Problem 6 (14 pts)

Suppose $\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4, \vec{x}_5, \vec{x}_6$ are 6 vectors in $\mathbb{R}^9$ such that

$$S = \text{span}\left(\{\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4, \vec{x}_5, \vec{x}_6\}\right)$$

is a **4-dimensional** subspace of $\mathbb{R}^9$.

a) (2 pts) True or False: The set $\{\vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4, \vec{x}_5, \vec{x}_6\}$ is linearly independent.
◯ True ◯ False

b) (4 pts) Consider the statement:

"There exists a vector $\vec{b} \in \mathbb{R}^9$ such that the number of ways to write $\vec{b}$ as a linear combination of $\vec{x}_1, \dots, \vec{x}_6$ is ___."

In each part below, a possible way to fill in the blank is given. Determine whether the statement that results from filling in the blank is **True** or **False**.

(i) zero
◯ True ◯ False

(ii) exactly one
◯ True ◯ False

(iii) exactly two
◯ True ◯ False

(iv) infinite
◯ True ◯ False

c) (4 pts) Suppose $\vec{b}$ is some vector in S such that both of the following equations are true:

$$\vec{b} = 4\vec{x}_1 - 2\vec{x}_2 + 6\vec{x}_3$$
$$\vec{b} = 3\vec{x}_1 + 3\vec{x}_3 - \vec{x}_5$$

State **one** other linear combination of $\vec{x}_1, \dots, \vec{x}_6$ that is equal to $\vec{b}$. Fill in each box with a number with no variables.

$\vec{b} = \square\, \vec{x}_1 + \square\, \vec{x}_2 + \square\, \vec{x}_3 + \square\, \vec{x}_4 + \square\, \vec{x}_5 + \square\, \vec{x}_6$

d) (4 pts) Let $T = \text{span}(\{\vec{x}_1, \vec{x}_2, \vec{x}_3\})$ and $U = \text{span}(\{\vec{x}_4, \vec{x}_5, \vec{x}_6\})$. Suppose W is the **intersection** of T and U, i.e. $W = T \cap U$. W is also a subspace of $\mathbb{R}^9$.

What are the smallest and largest possible values of dim(W), the dimension of W? Give your answers as integers.

smallest possible value $=$ $\square$ largest possible value $=$ $\square$

Problem 7 (20 pts)

Suppose we'd like to find the optimal constant parameter, w^*, for the constant model $h(x_i) = w$, given a dataset of n points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$. To do so, we use the **sub-squared** loss function, L_{ss}, defined below.

$$L_{\text{ss}}(y_i, w) = (\sqrt{y_i} - \sqrt{w})^2$$

This requires us to assume that all $y_i \geq 0$, as are all possible values of w.

a) (6 pts) Find $\frac{\mathrm{d}}{\mathrm{d}w} R_{\text{ss}}(w)$, the derivative of **average** sub-squared loss (i.e. the empirical risk) with respect to w. Show your work and circle your final answer, which should be an expression in terms of the y_i's, n, and/or any constants. *Hint: The derivative of* $f(x) = \sqrt{x}$ *is* $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$.

b) (6 pts) Show that the value of w^* that minimizes average sub-squared loss is

$$w^* = \left(\frac{1}{n} \sum_{i=1}^{n} \sqrt{y_i} \right)^2$$

c) (6 pts) Using the Cauchy-Schwarz inequality, prove that

$$\left(\frac{1}{n}\sum_{i=1}^{n}\sqrt{y_i}\right)^2 \leq \bar{y}$$

where $\bar{y}$ is the mean of the y_i's.

Solutions that do not use the Cauchy-Schwarz inequality will not receive credit.

d) (2 pts) What is the value of w that minimizes the following function:

$$R(w) = \frac{1}{n}\sum_{i=1}^{n}(y_i^4 - w^4)^2$$

Hint: This can be done without using any calculus — don't try and take the derivative.

○ $\left(\frac{1}{n}\sum_{i=1}^{n} y_i\right)^4$ ○ $\left(\frac{1}{n}\sum_{i=1}^{n} y_i^{1/4}\right)^4$ ○ $\left(\frac{1}{n}\sum_{i=1}^{n} y_i^4\right)^{1/4}$ ○ $\left(\frac{1}{n}\sum_{i=1}^{n} y_i^{1/2}\right)^{1/4}$ ○ $\left(\frac{1}{n}\sum_{i=1}^{n} y_i^4\right)^{1/2}$

Congrats on finishing Midterm 1!

Feel free to draw us a picture about EECS 245 in the box below (or use it for scratch work).