Homework 5: Linear Independence and Subspaces

EECS 245, Winter 2026 at the University of Michigan

due Friday, February 13th, 2026 at 11:59PM Ann Arbor Time

Write your solutions to the following problems by either typing them up or handwriting them on another piece of paper. Homeworks are due to Gradescope by 11:59PM on the due date. See the **syllabus** for details on the slip day policy.

Homework will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain and justify your conclusions, using sound reasoning. Your goal should be to convince the reader of your assertions. If a question does not require explanation, it will be explicitly stated.

Before proceeding, make sure you're familiar with the **collaboration policy**.

Total Points: 8 + 8 + 6 = 22

Problem 1: Linear Independence of New Vectors (8 pts)

Suppose $\vec{v}_1, \vec{v}_2, \vec{v}_3 \in \mathbb{R}^n$ **are linearly independent**. In both parts below, determine if the new set of vectors is linearly independent. If they are, prove that they are by showing that the only solution to the equation

$$a\vec{u}_1 + b\vec{u}_2 + c\vec{u}_3 = \vec{0}$$

is $a = b = c = 0$. If they are not, show that there exist scalars a, b, c such that $a\vec{u}_1 + b\vec{u}_2 + c\vec{u}_3 = \vec{0}$ where at least one of a, b, c is nonzero.

a) (4 pts) $\vec{u}_1 = \vec{v}_2 - \vec{v}_3$, $\vec{u}_2 = \vec{v}_1 - \vec{v}_3$, and $\vec{u}_3 = \vec{v}_1 - \vec{v}_2$

b) (4 pts) $\vec{u}_1 = \vec{v}_2 + \vec{v}_3$, $\vec{u}_2 = \vec{v}_1 + \vec{v}_3$, and $\vec{u}_3 = \vec{v}_1 + \vec{v}_2$

Problem 2: Thinking in Higher Dimensions (8 pts)

a) (3 pts) Suppose $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_8$ are 8 vectors in $\mathbb{R}^5$. Fill in each blank below with one of the provided options, and explain your reasoning.

(i) These vectors ________ span all of $\mathbb{R}^5$.
(options: do, do not, might)

(ii) These vectors ________ linearly independent.
(options: are, are not, might be)

(iii) Any 5 of these vectors ________ span all of $\mathbb{R}^5$.
(options: do, do not, might)

b) (5 pts) Suppose $\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_{10}$ are 10 non-zero vectors in $\mathbb{R}^{11}$.

Furthermore, suppose that $\text{span}(\{\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_{10}\})$ is a 6-dimensional subspace of $\mathbb{R}^{11}$. This means that there exists a subset of 6 of these vectors that is linearly independent and spans the same 6-dimensional subspace as the original 10 vectors; we just don't know which 6.

(i) Let k be the dimension of the subspace spanned by a subset of 4 of these vectors. What are all possible values of k?

(ii) Let m be the dimension of the subspace spanned by a subset of 7 of these vectors. What are all possible values of m?

Problem 3: Intersections of Subspaces (6 pts)

Let:

- M be the subspace of $\mathbb{R}^4$ spanned by $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -4 \\ 1 \\ 5 \end{bmatrix}$
- N be the subspace of $\mathbb{R}^4$ spanned by $\begin{bmatrix} 0 \\ -2 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \\ 1 \\ 3 \end{bmatrix}$

a) (2 pts) Find a vector that belongs to both M and N. (In other words, find a vector $\vec{v}$ such that $\vec{v} \in M$ and $\vec{v} \in N$.) There are infinitely many answers; state the answer with a first component of 1.

b) (4 pts) Fill in the blank and explain your reasoning: the set of all vectors that belong to both M and N is a subspace of $\mathbb{R}^4$ with dimension __________.