Lab 1: Python Basics and Math Review

EECS 245, Fall 2025 at the University of Michigan

due by the end of your lab section on Wednesday, August 27th, 2025

Name: ______________________________

uniqname: ______________________________

Welcome to the first lab of EECS 245!

Each lab worksheet will contain several activities, some of which will involve writing code and others that will involve writing math on paper. To receive credit for a lab, you must complete all activities and show your lab TA by the end of the lab section.

While you must get checked off by your lab TA **individually**, we encourage you to form groups with 1-2 other students to complete the activities together.

Activity 1: Guessing Game

We'll start with a math-related **icebreaker**. Your lab TA will explain how the activity works.

Context: a set of $n + 1$ points uniquely defines a polynomial of degree (at most) n.

- For example, the points $(2, 5)$ and $(5, 17)$ uniquely define the degree-1 polynomial

  $$y = 4x - 3$$

  By uniquely define, we mean that there is no other line that both of these points pass through.

- Similarly, the points $(1, 0)$, $(2, 1)$, and $(3, 6)$ uniquely define the degree-2 polynomial

  $$y = 2x^2 - 5x + 3$$

  There is a way of finding the equation of the specific degree-n polynomial that passes through $n + 1$ specific points, called interpolation. How it works is not important today.

  Key point: Given just two of these three points, it is impossible to recover this specific polynomial, because there are infinitely many degree-2 polynomials that pass through a pair of points.

So if one person writes down a polynomial of degree, say, 5, the other person needs to know 6 points on the polynomial to be able to discover it. 5 or 4 points won't suffice. This is a fact.

But, we're going to contradict this fact here! Partner 1 will guess Partner 2's polynomial by asking for just **2 points** on it, no matter the degree that Partner 2 selects. Have fun!

Activity 2: Environment Setup and Python Basics

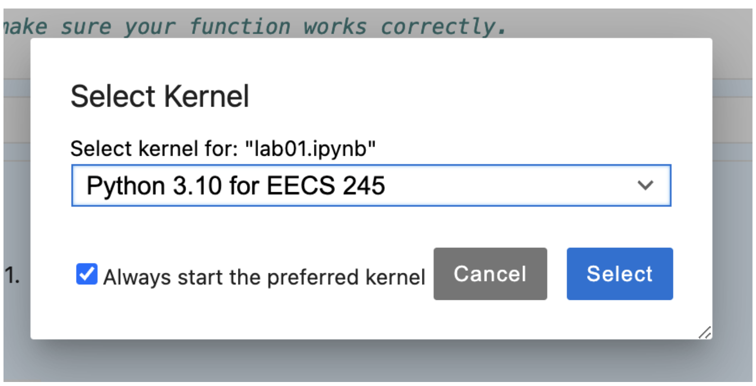
Labs and homeworks will both involve writing some Python code in a Jupyter Notebook. To access these Jupyter Notebooks (along with all necessary files and Python packages), you have two options:

- **Option 1:** Use DataHub — **datahub.eecs245.org** — a server we set up for this course, with all necessary packages pre-installed. Easier, but possibly slower.
- **Option 2:** Set up Jupyter Notebooks and necessary packages on your computer. Requires more setup.

Read the Tech Support section of the course website, **eecs245.org/tech-support**, for more details on the tradeoffs between both. To save time in lab, we'll start with Option 1.

In lab, all you need to do is click the "code" link under Lab 1 on the course website. This will prompt you to log in; use your uniqname and set a password.

Eventually, `lab01.ipynb` will open. Before you proceed, **click "Python 3 (ipykernel)" in the top right corner of the notebook** and select **"Python 3.10 for EECS 245"**. Make sure to click "Always start the preferred kernel"; you should only need to do this step once.



Then, you're ready to work on the lab! Read the notebook carefully, as it introduces the Python programming language and the Jupyter Notebook environment.

To receive credit for Activity 1, you'll need to submit your completed `lab01.ipynb` notebook to Gradescope and show your lab TA that all test cases have passed. Instructions on how to do this are in the lab notebook.

Activity 3: Running Mean

Over the summer, you ran a cherry pie stand. On days 1 through 5 (inclusive), you averaged 50 dollars per day in sales. On days 6 and 7, you averaged 22 dollars per day in sales. What were your average daily sales from days 1 through 7?

Solution: The key fact being assessed here is:

$$\text{mean} = \frac{\text{sum}}{\text{count}}$$

To find your new average daily sales, we need to find the sum of sales across all 7 days, and divide by the number of days (7).

- From days 1 to 5, you averaged 50 dollars per day, meaning your total sales from days 1 to 5 were $50 \cdot 5 = 250$ dollars.
- Similarly, your total sales from days 6 and 7 combined were $22 \cdot 2 = 44$ dollars.

So, your average daily sales across all 7 days is:

$$\frac{50 \cdot 5 + 22 \cdot 2}{7} = \frac{294}{7} = 42$$

Note that the first expression above can be written as:

$$\frac{5}{7} \cdot 50 + \frac{2}{7} \cdot 22$$

This is a **weighted average** or **weighted mean** of the numbers 50 and 22, with weights $\frac{5}{7}$ and $\frac{2}{7}$, respectively. Weighted averages appear all of the time in machine learning, but even in day-to-day life: your GPA is a weighted average of your grades in each class, where the weights are the number of credits earned.

Activity 4: A New Meaning

Over the summer, in addition to running your pie stand, you took a road trip to Chicago, 240 miles away.

a) For the first 120 miles, you averaged 80 miles per hour (mph). For the second 120 miles, you averaged 50 mph. What was your average speed throughout the entire journey? Leave your answer unsimplified in terms of fractions, but plug it into a calculator to get an approximation.

Solution: Following the same principle of mean $= \frac{\text{sum}}{\text{count}}$ from Activity 3, we have that:

$$\text{mean speed} = \frac{\text{total distance}}{\text{total time}}$$

The total distance traveled was 120 miles. What was the total time taken? We can break this up into time for Segment 1 + time for Segment 2.

- In Segment 1, we traveled 80 miles per hour for 120 miles, so:

$$80 \text{ miles per hour} = \frac{120 \text{ miles}}{\text{time for Segment 1}} \implies \text{time for Segment 1} = \frac{120}{80} \text{ hours}$$

- In Segment 2, we traveled 50 miles per hour for 120 miles, so:

$$\text{time for Segment 2} = \frac{120}{50} \text{ hours}$$

Putting this all together, we have:

$$\text{mean speed} = \frac{240 \text{ miles}}{\frac{120}{80} + \frac{120}{50} \text{ hours}}$$

Notice that both the numerator and denominator have a factor of 120. Pulling this out, we have:

$$\text{mean speed} = \frac{2}{\frac{1}{80} + \frac{1}{50}} \approx 61.54 \text{ miles per hour}$$

b) Suppose, instead, you drove 3 segments of 80 miles each, in which you averaged 80 mph, 80 mph, and 50 mph. What was your average speed throughout the entire journey?

Solution: Following the same pattern, we'd have:

$$\text{mean speed} = \frac{240 \text{ miles}}{\frac{80}{80} + \frac{80}{80} + \frac{80}{50} \text{ hours}} = \frac{3}{\frac{1}{80} + \frac{1}{80} + \frac{1}{50}} \approx 66.67 \text{ miles per hour}$$

c) In general, suppose you drove n segments of equal length, and averaged x_i mph in segment i ($i = 1, 2, ..., n$). What was your average speed throughout the entire journey? Give your answer using **summation notation**. Your answer is the formula for the **harmonic mean** of the numbers $x_1, x_2, ..., x_n$.

Solution: If we generalize the calculations from the previous two parts, we have:

$$\begin{aligned}
\text{mean speed} &= \frac{240}{\frac{\frac{240}{n}}{x_1} + \frac{\frac{240}{n}}{x_2} + \ldots + \frac{\frac{240}{n}}{x_n}} \\
&= \frac{240}{\frac{240}{n}\left(\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}\right)} \\
&= \frac{1}{\frac{1}{n}\left(\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}\right)} \\
&= \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}} \\
&= \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}
\end{aligned}$$

This formula computes the **harmonic mean** of the numbers $x_1, x_2, ..., x_n$. Notice that 240 doesn't appear in the final answer.

Activity 5: The Meaning of Calculus

Here, we'll review key ideas from Calculus 1. If you'd like a refresher, see **Chapter 0.2** of the course notes, **notes.eecs245.org**.

Consider the function:

$$f(x) = (x-3)^2 + (x-4)^2 + (x-5)^2 + (x-16)^2$$

a) What is the shape of $f(x)$? Your answer should be a single word.

> **Solution:**
> $f(x)$ is a quadratic function, i.e. a parabola. We're not sure where it's centered yet — that's the goal of parts (b) and (c).

b) Find $\frac{\mathrm{d}f}{\mathrm{d}x}$, the derivative of $f(x)$.

> **Solution:**
>
> $$\begin{aligned}\frac{\mathrm{d}f}{\mathrm{d}x} &= \frac{\mathrm{d}}{\mathrm{d}x}\left((x-3)^2 + (x-4)^2 + (x-5)^2 + (x-16)^2\right) \\ &= 2(x-3) + 2(x-4) + 2(x-5) + 2(x-16)\end{aligned}$$

c) Find x^*, the value of x that minimizes $f(x)$, and prove that it is indeed a minimum, rather than a maximum.

Solution:
First, we'll set the derivative we found in part (b) to 0:

$$2(x-3)+2(x-4)+2(x-5)+2(x-16)=0$$

We can divide both sides by 2:

$$x-3+x-4+x-5+x-16=0$$

Finally, we have:

$$4x=(3+4+5+16) \implies \boxed{x^*=\frac{3+4+5+16}{4}=7}$$

To show that x^* is indeed a minimum, we need to show that the second derivative of $f(x)$ is positive at x^*.

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2}=\frac{\mathrm{d}}{\mathrm{d}x}\left(2(x-3)+2(x-4)+2(x-5)+2(x-16)\right)=2+2+2+2=8$$

Since the second derivative is positive everywhere, $f(x)$ is a convex function, and therefore has a global minimum at x^*.

d) What does the value of x^* have to do with the numbers 3, 4, 5, and 16?

Solution:
x^* is the mean of the numbers 3, 4, 5, and 16.

$$4x=(3+4+5+16) \implies x^*=\frac{3+4+5+16}{4}=7$$

$$f(x) = (x-3)^2 + (x-4)^2 + (x-5)^2 + (x-16)^2$$

e) For each of the following functions $g(x)$, identify all extrema (that is, maximums and/or minimums). You don't need to take the derivative in each case, but explain your reasoning.

1. $g(x) = \frac{1}{4}f(x)$

Solution:
$g(x)$ is minimized at $x^* = 7$.

$g(x)$ has the same vertex as $f(x)$, but it is scaled vertically by a factor of $\frac{1}{4}$.

If that's not convincing, note that the derivative of $g(x)$ is just $\frac{1}{4}$ times the derivative of $f(x)$. When we set the derivative of $g(x)$ to 0, we'll end up solving the same equation for x^* as we did in part (c):

$$\begin{aligned} \frac{\mathrm{d}g}{\mathrm{d}x} &= 0 \\ \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{4}f(x)\right) &= 0 \\ \frac{1}{4}\frac{\mathrm{d}f}{\mathrm{d}x} &= 0 \\ \frac{\mathrm{d}f}{\mathrm{d}x} &= 0 \end{aligned}$$

2. $g(x) = -5f(x)$

Solution:
$g(x)$ is maximized at $x^* = 7$.

$g(x)$ does not have a global minimum — it is a downward-facing parabola, since $f(x)$ is an upward-facing parabola, and $g(x)$ is just $f(x)$ scaled by a factor of -5.

Following the same logic as in the previous subpart, we know that the derivative of $-5f(x)$ is also 0 at $x^* = 7$. The difference is that $x^* = 7$ corresponds to a global maximum, rather than a global minimum.

3. $g(x) = f(2x)$

Solution:
$g(x)$ is minimized at $x^* = \frac{7}{2}$.

$g(x)$ is also an upward-facing parabola, but it is compressed horizontally by a factor of 2. It's a little more difficult to reason about horizontal compressions, so let's work through the derivative:

$$\begin{aligned}
\frac{\mathrm{d}g}{\mathrm{d}x} &= \frac{\mathrm{d}}{\mathrm{d}x}\left(f(2x)\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left((2x-3)^2 + (2x-4)^2 + (2x-5)^2 + (2x-16)^2\right) \\
&= 2(2x-3)\cdot 2 + 2(2x-4)\cdot 2 + 2(2x-5)\cdot 2 + 2(2x-16)\cdot 2 \\
&= 4(2x-3) + 4(2x-4) + 4(2x-5) + 4(2x-16)
\end{aligned}$$

In the second-last line above, the additional factors of two are the result of the chain rule (the derivative of $2x-3$ with respect to x is 2), and you'll notice that each term in parentheses involves $2x$, not just x as with $f(x)$.

Setting the derivative of $g(x)$ to 0, we have:

$$\begin{aligned}
4(2x-3) + 4(2x-4) + 4(2x-5) + 4(2x-16) &= 0 \\
2x - 3 + 2x - 4 + 2x - 5 + 2x - 16 &= 0 \\
8x &= 3 + 4 + 5 + 16 \\
x^* &= \frac{3+4+5+16}{8} = \frac{7}{2}
\end{aligned}$$

Intuitively, if we set $u = 2x$, then $g(x) = f(u)$, and we know that $f(u)$ is minimized at $u^* = 7$. Since $u = 2x$, we have $x^* = \frac{u^*}{2} = \frac{7}{2}$.

4. $g(x) = \sqrt{f(x)}$

Solution: $g(x)$ is minimized at $x^* = 7$.

$\sqrt{x}$ is a **strictly monotonically increasing** function across its domain, which is $[0, \infty)$. What this means is that if $a > b$, then $\sqrt{a} > \sqrt{b}$, or in other words, as we move from left to right, the graph of the function only increases, never stays the same or decreases. Strictly monotonically increasing functions preserve the order of their inputs.

$\log(x)$ is also a strictly monotonically increasing function. x^2 is **not**, because, for example, $(-3)^2 > 2^2$, but -3 is not greater than 2.

What does this have to do with finding the extrema of $g(x)$? Well, since we know that $f(7)$ is at the bottom of the graph of $f(x)$, we know that $\sqrt{f(7)}$ is at the bottom of the graph of $\sqrt{f(x)}$, because of the fact that $\sqrt{x}$ is strictly monotonically increasing, meaning that order is preserved.

If that's not a convincing argument, we can also work through the derivative:

$$\begin{aligned}
\frac{\mathrm{d}g}{\mathrm{d}x} &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{f(x)}\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)\right)^{\frac{1}{2}} \\
&= \frac{1}{2}\left(f(x)\right)^{-\frac{1}{2}} \cdot \frac{\mathrm{d}f}{\mathrm{d}x} \\
&= \frac{1}{2\sqrt{f(x)}} \cdot \frac{\mathrm{d}f}{\mathrm{d}x}
\end{aligned}$$

To solve for the extrema of $g(x)$, we need to set its derivative to 0. Its derivative contains two factors, one of which is $\frac{\mathrm{d}f}{\mathrm{d}x}$ — which we know is 0 at $x^* = 7$ — and the other is $\frac{1}{2\sqrt{f(x)}}$, which can never be 0 (think about why). So, $g(x)$ must be minimized at $x^* = 7$.
Visualize $f(x)$ and $g(x)$ **here** on Desmos.

5. $g(x) = f(x) + cx^2$, where $c \in \mathbb{R}$ (Hint: This may take more effort than the previous 4 did.)

Solution:
$g(x)$ is minimized at $x^* = \frac{28}{4+c}$.

It's hard to reason about the extrema of $g(x)$ without taking the derivative, at least at first.

$$\begin{aligned}\frac{\mathrm{d}g}{\mathrm{d}x} &= \frac{\mathrm{d}}{\mathrm{d}x}\left(f(x) + cx^2\right) \\ &= \frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)\right) + \frac{\mathrm{d}}{\mathrm{d}x}\left(cx^2\right) \\ &= 2(x-3) + 2(x-4) + 2(x-5) + 2(x-16) + 2cx\end{aligned}$$

Setting the derivative to 0, we have:

$$\begin{aligned}2(x-3) + 2(x-4) + 2(x-5) + 2(x-16) + 2cx &= 0 \\ x - 3 + x - 4 + x - 5 + x - 16 + cx &= 0 \\ (4+c)x &= 3 + 4 + 5 + 16\end{aligned}$$

So:

$$x^* = \frac{3+4+5+16}{4+c} = \boxed{\frac{28}{4+c}}$$

You'll notice that if $c = 0$, then $x^* = 7$, which is the same as the minimum of $f(x)$, as this equates to "turning off" the new cx^2 term.

How else could we have reasoned about x^*? One way to think about it is that Mean$(x_1, x_2, ..., x_n)$ minimizes:

$$(x - x_1)^2 + (x - x_2)^2 + \ldots + (x - x_n)^2$$

This is a generalization of your discovery from part (d), where $x_1, x_2, ..., x_n$ were the numbers 3, 4, 5, and 16.

When we added cx^2 to $f(x)$, it was almost like adding the value x^2, or equivalently, $(x-0)^2$, c times. In other words:

$$g(x) = (x-3)^2 + (x-4)^2 + (x-5)^2 + (x-16)^2 + \underbrace{x^2 + x^2 + \ldots + x^2}_{c \text{ times}}$$

So, knowing that the mean minimizes the sum of squared errors, we can conclude that x^* should be the mean of the numbers $3, 4, 5, 16, 0, ..., 0$ (where there are c zeros). The mean of these numbers is their sum over their count; their sum is $3 + 4 + 5 + 16 + 0 + ... + 0 = 28$, and their count is $4 + c$. So, $x^* = \frac{28}{4+c}$.

This logic is not perfect, since c didn't need to be an integer, but it helps build intuition for the answer.

Activity 6: Manipulating Sums

Here, we'll review the basics of summation notation. If you'd like a refresher, see **Chapter 0.1** of the course notes, **notes.eecs245.org**.

Consider the following summations involving the first n natural numbers, $1, 2, 3, ..., n$.

$$1 + 2 + 3 + \ldots + n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$
$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Using the formulas above, determine the values of each of the following sums.

a) $\sum_{i=5}^{15} i^2$

Solution:
The key is recognizing that we can express the sum we're looking for as the difference of two other sums that have closed-form expressions:

$$\sum_{i=5}^{15} i^2 = \sum_{i=1}^{15} i^2 - \sum_{i=1}^{4} i^2$$

Given that, we have:

$$\begin{aligned}\sum_{i=5}^{15} i^2 &= \sum_{i=1}^{15} i^2 - \sum_{i=1}^{4} i^2 \\ &= \frac{15(16)(31)}{6} - \frac{4(5)(9)}{6} \\ &= 1240 - 30 \\ &= 1210\end{aligned}$$

b) $\sum_{i=4}^{9} 3$

Solution:
Notice that the sum we're looking for is just 3 added together, several times — it does not involve i.

How many times are we adding 3? It's tempting to think it's 5 times, since $9 - 4 = 5$, but that is one short. Count out the numbers from 4 to 9 to see that the range includes 6 numbers, when we include both endpoints.

So, the sum is:

$$\sum_{i=4}^{9} 3 = 3 + 3 + 3 + 3 + 3 + 3 = 3 \cdot (9 - 4 + 1) = 3 \cdot 6 = 18$$

c) $\sum_{k=0}^{12} (k+2)$

Solution:
We can separate the sum into two smaller sums:

$$\sum_{k=0}^{12} (k+2) = \sum_{k=0}^{12} k + \sum_{k=0}^{12} 2$$

The first sum, $\sum_{k=0}^{12} k$, is the sum of the first 12 natural numbers, which is $\frac{12 \cdot 13}{2} = 78$. The fact that we started from $k = 0$ instead of $k = 1$ doesn't change anything here, since adding 0 does not affect the sum.

The second sum, $\sum_{k=0}^{12} 2$, is just 2 added together, 13 times, which is $2 \cdot 13 = 26$. Here, the fact we started from $k = 0$ instead of $k = 1$ does actually matter.

So, the full sum is:

$$\sum_{k=0}^{12} (k+2) = \sum_{k=0}^{12} k + \sum_{k=0}^{12} 2 = 78 + 26 = 104$$

Another way of arriving at the solution is to recognize that:

$$\sum_{k=0}^{12} (k+2) = \sum_{k=2}^{14} k = \left(\sum_{k=1}^{14} k \right) - 1 = \frac{14 \cdot 15}{2} - 1 = 104$$

d) $\sum_{j=1}^{20} (1 - 3j)^2$

Solution:
We'll have to expand a fair bit here:

$$\begin{aligned}\sum_{j=1}^{20}(1-3j)^2 &= \sum_{j=1}^{20}(1-6j+9j^2)\\ &= \sum_{j=1}^{20}1-\sum_{j=1}^{20}6j+\sum_{j=1}^{20}9j^2\\ &= 20-6\sum_{j=1}^{20}j+9\sum_{j=1}^{20}j^2\end{aligned}$$

We know that $\sum_{j=1}^{20} j = \frac{20\cdot 21}{2} = 210$ and $\sum_{j=1}^{20} j^2 = \frac{20\cdot 21\cdot 41}{6} = 2870$. So, we have:

$$\begin{aligned}\sum_{j=1}^{20}(1-3j)^2 &= 20-6\cdot 210+9\cdot 2870\\ &= 20-1260+25830\\ &= 24590\end{aligned}$$