

# Lab 1: Math Foundations and Environment Setup

EECS 245, Winter 2026 at the University of Michigan

due by the end of your lab section

Name: \_\_\_\_\_

username: \_\_\_\_\_

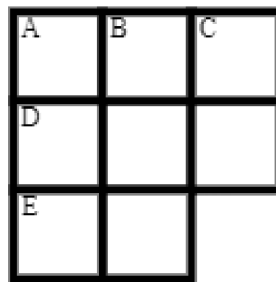
Welcome to the first lab of EECS 245!

Each lab worksheet will contain several activities, some of which will involve writing code and others that will involve writing math on paper. To receive credit for a lab, you must complete all activities and show your lab TA by the end of the lab section.

While you must get checked off by your lab TA **individually**, we encourage you to form groups with 1-2 other students to complete the activities together.

## Activity 1: Icebreaker: Cross-Number (Source: Berkeley Math Tournament 2023)

Take a moment to introduce yourself to the person sitting next to you and exchange contact information. Then, together, solve the following cross-number puzzle. Each spot in the grid should be filled with a digit from 0 to 9, using the clues below. Digits may be repeated.



- A Across: A number with only even digits, in strictly descending order (3)
- D Across: A number not divisible by 9 (3)
- E Across: A number divisible by 11 (2)
- A Down: A number with consecutive digits, in ascending order (3)
- B Down: A number where the product of the lesser-valued two digits is equal to the largest digit (3)
- C Down: A prime number greater than 10 (2)

### Activity 2: Running Mean

Over the break, you ran a hot chocolate stand. On days 1 through 5 (inclusive), you averaged 50 dollars per day in sales. On days 6 and 7, you averaged 22 dollars per day in sales. What were your average daily sales from days 1 through 7?

### Activity 3: A New Meaning

Over the break, in addition to running your hot chocolate stand, you took a road trip to Chicago, 240 miles away.

- a) For the first 120 miles, you averaged 80 miles per hour (mph). For the second 120 miles, you averaged 50 mph. What was your average speed throughout the entire journey? Leave your answer unsimplified in terms of fractions, but plug it into a calculator to get an approximation.

- b) Suppose, instead, you drove 3 segments of 80 miles each, in which you averaged 80 mph, 80 mph, and 50 mph. What was your average speed throughout the entire journey?

- c) In general, suppose you drove  $n$  segments of equal length, and averaged  $x_i$  mph in segment  $i$  ( $i = 1, 2, \dots, n$ ). What was your average speed throughout the entire journey? Give your answer using **summation notation**. Your answer is the formula for the **harmonic mean** of the numbers  $x_1, x_2, \dots, x_n$ .

#### Activity 4: The Meaning of Calculus

Here, we'll review key ideas from Calculus 1. If you'd like a refresher, see [Appendix 2](#) of the course notes, [notes.eecs245.org](https://notes.eecs245.org).

Consider the function:

$$f(x) = (x - 3)^2 + (x - 4)^2 + (x - 5)^2 + (x - 16)^2$$

- a) What is the shape of  $f(x)$ ? Your answer should be a single word.

- b) Find  $\frac{df}{dx}$ , the derivative of  $f(x)$ .

- c) Find  $x^*$ , the value of  $x$  that minimizes  $f(x)$ , and prove that it is indeed a minimum, rather than a maximum.

- d) What does the value of  $x^*$  have to do with the numbers 3, 4, 5, and 16?

### Activity 5: Basics of Summation Notation

Here, we'll review the basics of summation notation. If you'd like a refresher, see [Appendix 1](#) of the course notes, [notes.eecs245.org](https://notes.eecs245.org).

Consider the following formula involving the first  $n$  natural numbers,  $1, 2, \dots, n$ .

$$1 + 2 + 3 + \dots + n = \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

Using the fact above, find  $\sum_{k=4}^{12} (k+2)$ . Verify your answer by calculating the sum directly.

### Activity 6: Environment Setup and Python Basics

Labs and homeworks will both involve writing some Python code in a Jupyter Notebook. To access these Jupyter Notebooks (along with all necessary files and Python packages), you have two options:

- **Option 1:** Set up Jupyter Notebooks and necessary packages on your computer. Requires more setup, but is more robust.
- **Option 2:** Use DataHub — [datahub.eecs245.org](https://datahub.eecs245.org) — a server we set up for this course, with all necessary packages pre-installed. Easier, but less reliable and slower.

Read the Environment Setup section of the course website, [eecs245.org/env-setup](https://eecs245.org/env-setup), for detailed steps on setting up a local environment on your machine. Take the time to follow the steps under **Option 1: Local Setup**, and let us know if you have any questions.

Then, open the notebook `labs/lab01/lab01.ipynb` locally, read it, and complete the tasks inside. Once you're done, let your TA know that you're ready to be checked off.

The rest of this worksheet is extra practice. Don't feel pressured to answer all of these problems in lab, but make sure to attempt them at some point.

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### Activity 7: The Meaning of Calculus, Continued

$$f(x) = (x - 3)^2 + (x - 4)^2 + (x - 5)^2 + (x - 16)^2$$

For each of the following functions  $g(x)$ , identify all extrema (that is, maximums and/or minimums). You don't need to take the derivative in each case, but explain your reasoning.

a)  $g(x) = \frac{1}{4}f(x)$

b)  $g(x) = -f(2x)$

c)  $g(x) = \sqrt{f(x)}$

d)  $g(x) = f(x) + cx^2$ , where  $c \in \mathbb{R}$  (Hint: This may take more effort than the previous 4 did.)

### Activity 8: Summation Notation Properties

Suppose  $x_1, x_2, \dots, x_n$  and  $y_1, y_2, \dots, y_n$  are both lists of numbers. Determine whether each of the following expressions is true or false.

a)  $\sum_{i=1}^n (ax_i + b) = a \sum_{i=1}^n x_i + bn$ , where  $a$  and  $b$  are constants.

b)  $\sum_{i=1}^n (x_i + y_i)^2 = \sum_{i=1}^n x_i^2 + \sum_{i=1}^n y_i^2$

c)  $\sum_{i=2}^n x_i = \sum_{i=2}^k x_i + \sum_{i=k}^n x_i$

d)  $\sum_{i=1}^n (x_i - \bar{x}) = \sum_{i=1}^n x_i - n\bar{x}$ , where  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$

### Activity 9: Manipulating Sums

Consider the following summations involving the first  $n$  natural numbers,  $1, 2, 3, \dots, n$ .

$$1 + 2 + 3 + \dots + n = \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

Using the formulas above, determine the values of each of the following sums.

a)  $\sum_{i=5}^{15} i^2$

b)  $\sum_{i=4}^9 3$

c)  $\sum_{j=1}^{20} (1 - 3j)^2$