Lab 1: Python Basics and Math Review

EECS 245, Fall 2025 at the University of Michigan

due by the end of your lab section on Wednesday, August 27th, 2025

Name: ______________________________

uniqname: ______________________________

Welcome to the first lab of EECS 245!

Each lab worksheet will contain several activities, some of which will involve writing code and others that will involve writing math on paper. To receive credit for a lab, you must complete all activities and show your lab TA by the end of the lab section.

While you must get checked off by your lab TA **individually**, we encourage you to form groups with 1-2 other students to complete the activities together.

Activity 1: Guessing Game

We'll start with a math-related **icebreaker**. Your lab TA will explain how the activity works.

Context: a set of $n+1$ points uniquely defines a polynomial of degree (at most) n.

- For example, the points $(2,5)$ and $(5,17)$ uniquely define the degree-1 polynomial

  $$y = 4x - 3$$

  By uniquely define, we mean that there is no other line that both of these points pass through.

- Similarly, the points $(1,0)$, $(2,1)$, and $(3,6)$ uniquely define the degree-2 polynomial

  $$y = 2x^2 - 5x + 3$$

  There is a way of finding the equation of the specific degree-n polynomial that passes through $n+1$ specific points, called interpolation. How it works is not important today.

  Key point: Given just two of these three points, it is impossible to recover this specific polynomial, because there are infinitely many degree-2 polynomials that pass through a pair of points.

So if one person writes down a polynomial of degree, say, 5, the other person needs to know 6 points on the polynomial to be able to discover it. 5 or 4 points won't suffice. This is a fact.

But, we're going to contradict this fact here! Partner 1 will guess Partner 2's polynomial by asking for just **2 points** on it, no matter the degree that Partner 2 selects. Have fun!

Activity 2: Environment Setup and Python Basics

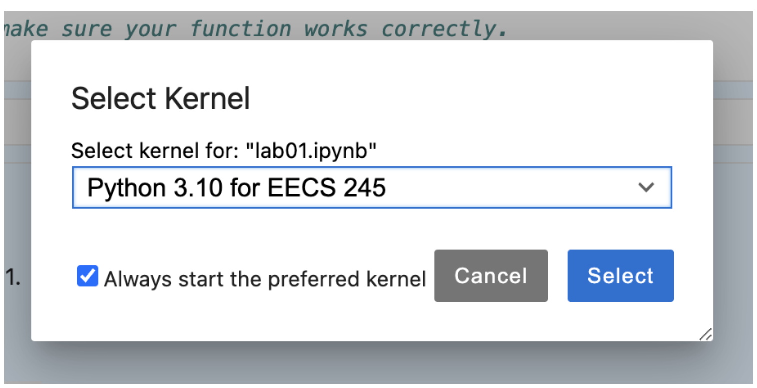
Labs and homeworks will both involve writing some Python code in a Jupyter Notebook. To access these Jupyter Notebooks (along with all necessary files and Python packages), you have two options:

- **Option 1:** Use DataHub — **datahub.eecs245.org** — a server we set up for this course, with all necessary packages pre-installed. Easier, but possibly slower.
- **Option 2:** Set up Jupyter Notebooks and necessary packages on your computer. Requires more setup.

Read the Tech Support section of the course website, **eecs245.org/tech-support**, for more details on the tradeoffs between both. To save time in lab, we'll start with Option 1.

In lab, all you need to do is click the "code" link under Lab 1 on the course website. This will prompt you to log in; use your uniqname and set a password.

Eventually, `lab01.ipynb` will open. Before you proceed, **click "Python 3 (ipykernel)" in the top right corner of the notebook** and select **"Python 3.10 for EECS 245"**. Make sure to click "Always start the preferred kernel"; you should only need to do this step once.



Then, you're ready to work on the lab! Read the notebook carefully, as it introduces the Python programming language and the Jupyter Notebook environment.

To receive credit for Activity 1, you'll need to submit your completed `lab01.ipynb` notebook to Gradescope and show your lab TA that all test cases have passed. Instructions on how to do this are in the lab notebook.

Activity 3: Running Mean

Over the summer, you ran a cherry pie stand. On days 1 through 5 (inclusive), you averaged 50 dollars per day in sales. On days 6 and 7, you averaged 22 dollars per day in sales. What were your average daily sales from days 1 through 7?

Activity 4: A New Meaning

Over the summer, in addition to running your pie stand, you took a road trip to Chicago, 240 miles away.

a) For the first 120 miles, you averaged 80 miles per hour (mph). For the second 120 miles, you averaged 50 mph. What was your average speed throughout the entire journey? Leave your answer unsimplified in terms of fractions, but plug it into a calculator to get an approximation.

b) Suppose, instead, you drove 3 segments of 80 miles each, in which you averaged 80 mph, 80 mph, and 50 mph. What was your average speed throughout the entire journey?

c) In general, suppose you drove n segments of equal length, and averaged x_i mph in segment i ($i = 1, 2, ..., n$). What was your average speed throughout the entire journey? Give your answer using **summation notation**. Your answer is the formula for the **harmonic mean** of the numbers $x_1, x_2, ..., x_n$.

Activity 5: The Meaning of Calculus

Here, we'll review key ideas from Calculus 1. If you'd like a refresher, see **Chapter 0.2** of the course notes, **notes.eecs245.org**.

Consider the function:

$$f(x) = (x-3)^2 + (x-4)^2 + (x-5)^2 + (x-16)^2$$

a) What is the shape of $f(x)$? Your answer should be a single word.

b) Find $\frac{\mathrm{d}f}{\mathrm{d}x}$, the derivative of $f(x)$.

c) Find x^*, the value of x that minimizes $f(x)$, and prove that it is indeed a minimum, rather than a maximum.

d) What does the value of x^* have to do with the numbers 3, 4, 5, and 16?

$$f(x) = (x-3)^2 + (x-4)^2 + (x-5)^2 + (x-16)^2$$

e) For each of the following functions $g(x)$, identify all extrema (that is, maximums and/or minimums). You don't need to take the derivative in each case, but explain your reasoning.

1. $g(x) = \frac{1}{4}f(x)$

2. $g(x) = -5f(x)$

3. $g(x) = f(2x)$

4. $g(x) = \sqrt{f(x)}$

5. $g(x) = f(x) + cx^2$, where $c \in \mathbb{R}$ (Hint: This may take more effort than the previous 4 did.)

Activity 6: Manipulating Sums

Here, we'll review the basics of summation notation. If you'd like a refresher, see **Chapter 0.1** of the course notes, **notes.eecs245.org**.

Consider the following summations involving the first n natural numbers, $1, 2, 3, ..., n$.

$$1 + 2 + 3 + \ldots + n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Using the formulas above, determine the values of each of the following sums.

a) $\sum_{i=5}^{15} i^2$

b) $\sum_{i=4}^{9} 3$

c) $\sum_{k=0}^{12} (k+2)$

d) $\sum_{j=1}^{20} (1-3j)^2$