Lab 4: Vectors and the Dot Product

EECS 245, Winter 2026 at the University of Michigan

due by the end of your lab section

Name: ______________________________

uniqname: ______________________________

Each lab worksheet will contain several activities, some of which will involve writing code and others that will involve writing math on paper. To receive credit for a lab, you must complete all activities and show your lab TA by the end of the lab section.

While you must get checked off by your lab TA **individually**, we encourage you to form groups with 1-2 other students to complete the activities together.

Recap: Vectors and the Dot Product

- (Chapters 3.1 and 3.2) The **norm** of a vector $\vec{v} \in \mathbb{R}^n$ measures its length:
$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$$
This is the default norm for vectors in $\mathbb{R}^n$, but other norms exist.
- (3.1) A **linear combination** of the vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_d$ is any vector that can be written as
$$a_1\vec{v}_1 + a_2\vec{v}_2 + \cdots + a_d\vec{v}_d$$
where $a_1, a_2, \ldots, a_d$ are scalars. We can think of this as taking bits of each vector and adding them together. The a_i's are called the **coefficients** of the linear combination.
- (3.3) The **dot product** of two vectors $\vec{u}, \vec{v} \in \mathbb{R}^n$ is defined as:
$$\vec{u} \cdot \vec{v} = \begin{bmatrix} u_1 \\ u_2 \\ \cdots \\ u_n \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ \cdots \\ v_n \end{bmatrix} = u_1v_1 + u_2v_2 + \cdots + u_nv_n$$
The result is a **scalar**, not another vector.
- (3.3) The dot product also has a geometric definition, involving the norms (lengths) of the vectors and the angle between them:
$$\vec{u} \cdot \vec{v} = ||\vec{u}||||\vec{v}||\cos\theta$$
- (3.3) The key takeaway from the dot product is that it tells us how similar the directions of two vectors are. When two vectors have a dot product of 0, they are **orthogonal**, or have a 90 degree angle between them.

Activity 1: Linear Combinations

Let $\vec{u} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} -6 \\ 9 \end{bmatrix}$.

a) Find values of a and b such that $a\vec{u} + b\vec{v} = \vec{w}$. By finding a and b, you have written $\vec{w}$ as a **linear combination** of $\vec{u}$ and $\vec{v}$.

b) Now, try and write $\vec{w}$ as a linear combination of $\vec{u}$, $\vec{v}$, and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$. In other words, try and find values of a, b, and c such that

$$a \begin{bmatrix} 4 \\ 3 \end{bmatrix} + b \begin{bmatrix} -1 \\ -3 \end{bmatrix} + c \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \vec{w}$$

What happens? Why?

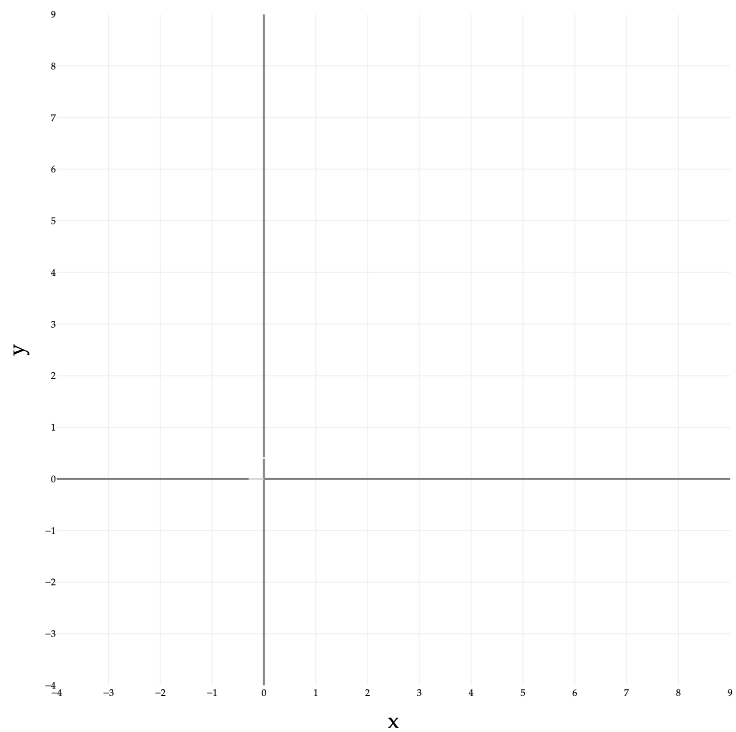
c) Now, try and write $\vec{w}$ as a linear combination of $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -4 \\ -2 \end{bmatrix}$. What happens? Why?

Activity 2: The Dot Product

For each pair of vectors below (1) draw them on the grid at the bottom of the page and (2) compute their dot product.

a) $\begin{bmatrix} 8 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 8 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} -5 \\ 0 \end{bmatrix}$



c) $\begin{bmatrix} 8 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 8 \end{bmatrix}$

d) $\begin{bmatrix} 8 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} 8 \\ 6 \end{bmatrix}$

e) $\begin{bmatrix} 8 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 4 \end{bmatrix}$

Activity 3: Angles and Orthogonality

In this activity, we will investigate the relationship between the two definitions of the dot product and learn how to use this equivalence to measure the similarity between two vectors.

$$\vec{u} \cdot \vec{v} = \begin{bmatrix} u_1 \\ u_2 \\ \dots \\ u_n \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ \dots \\ v_n \end{bmatrix} = u_1v_1 + u_2v_2 + \cdots + u_nv_n$$

$$\vec{u} \cdot \vec{v} = ||\vec{u}||||\vec{v}||\text{cos}\theta$$

Let $\vec{w} = \begin{bmatrix} 5 \\ 0 \\ -4 \\ 1 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} 9 \\ 1 \\ 2 \\ 3 \end{bmatrix}$.

a) Find $\vec{w} \cdot \vec{x}$, $\|\vec{w}\|$, and $\|\vec{x}\|$.

b) Using the results of part **a)**, find the angle between $\vec{w}$ and $\vec{x}$. Leave your answer in the form $\cos^{-1}(\cdot)$.

c) What is $\cos(90°)$? What does this have to do with orthogonality?

Activity 4: Sum–Difference Orthogonality

Let $\vec{u} = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 5 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 4 \\ -3 \end{bmatrix}$.

a) Show that $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are orthogonal.

b) Now suppose $\vec{u}, \vec{v} \in \mathbb{R}^n$ are arbitrary vectors with the same number of components. Is it always true that $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are orthogonal?

- If so, prove why.
- If not, specify conditions under which it's guaranteed that $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are orthogonal.

Hint: Use the distributive property of the dot product, which states that

$$(\vec{a} + \vec{b}) \cdot (\vec{c} + \vec{d}) = \vec{a} \cdot \vec{c} + \vec{a} \cdot \vec{d} + \vec{b} \cdot \vec{c} + \vec{b} \cdot \vec{d}$$

Activity 5: Triangle Inequality

The triangle inequality states that for any two vectors $\vec{u}, \vec{v} \in \mathbb{R}^n$:

$$\|\vec{u} + \vec{v}\| \leq \|\vec{u}\| + \|\vec{v}\|$$

a) **For the vectors** $\vec{u} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ **and** $\vec{v} = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$, verify that the triangle inequality holds. That is, show that the left-hand side is less than or equal to the right-hand side.

b) Find two **different** vectors in $\vec{x}, \vec{y} \in \mathbb{R}^2$ such that the triangle inequality achieves **equality**, i.e. where

$$\|\vec{x} + \vec{y}\| = \|\vec{x}\| + \|\vec{y}\|$$

What is the relationship between the $\vec{x}$ and $\vec{y}$ you found?

Activity 6: Arrays in NumPy

Instead of writing code in a separate Jupyter Notebook for this lab, you will interact with the code cells that exist in the course notes.

In particular, go to Chapter 3.2 of the course notes, scroll all the way to the bottom, and complete **Activity 5** there. To get checked off, show your lab TA that you've completed the activity — there's no need to submit your code anywhere.