## Lab 7: Inverses

EECS 245, Fall 2025 at the University of Michigan **due** by the end of your lab section on Wednesday, October 8th, 2025

Name:
uniqname:
Each lab worksheet will contain several activities, some of which will involve writing code and others that will involve writing math on paper. To receive credit for a lab, you must complete all activities and show your lab TA by the end of the lab section.
While you must get checked off by your lab TA <b>individually</b> , we encourage you to form groups with 1-2 other students to complete the activities together.
Activity 1: Basics of Invertibility
Suppose $A$ is an $n \times n$ matrix. Chapter 2.9 describes several equivalent conditions that guarantee that $A$ is invertible. State as many of these equivalent conditions as you can, <b>without</b> looking at the notes.
Activity 2: Symbolic Inverses
Given that $A$ is an invertible $n \times n$ matrix that satisfies $A^4 - 3A^2 + 2A - 4I = 0$ , find an expression for $A^{-1}$ in terms of $A$ .

## **Activity 3: True or False?**

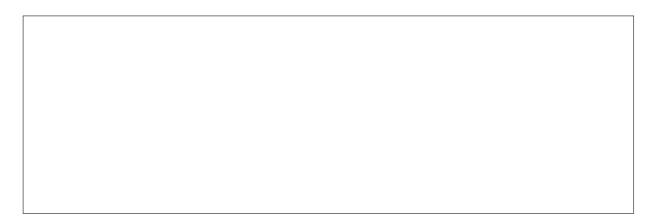
In each part, either prove that the statement is true or provide a counterexample.

**a)** If *A* and *B* are both invertible  $n \times n$  matrices, then A + B is invertible.

**b)** If  $A^2$  is invertible, then A is invertible.



c) If  $A \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ 3 \end{bmatrix}$  and  $A \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ , then A is invertible. (What could rank(A) be?)



## Activity 4: The $2 \times 2$ Case

Recall that the inverse of the  $2 \times 2$  matrix  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  is given by

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Using the fact above, find scalars  $x_1$  and  $x_2$  such that

$$2x_1 - 3x_2 = 6$$

$$5x_1 + 5x_2 = 10$$

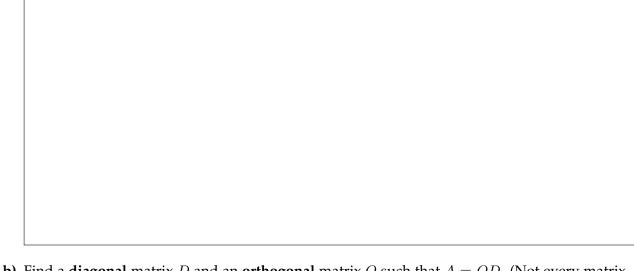
Hint: First, write the system of equations in the form  $A\vec{x} = \vec{b}$ . If A is invertible, and  $A\vec{x} = \vec{b}$ , then what is  $\vec{x}$ ?

## **Activity 5: Thinking in Transformations**

Suppose  $f: \mathbb{R}^3 \to \mathbb{R}^3$  is a linear transformation represented by the matrix A.

Furthermore, suppose that  $f\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}0\\3\\4\end{bmatrix}$ ,  $f\left(\begin{bmatrix}0\\10\\0\end{bmatrix}\right) = \begin{bmatrix}0\\4\\-3\end{bmatrix}$ , and  $f\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}1\\0\\0\end{bmatrix}$ .

a) Find  $f\left(\begin{bmatrix}2\\1\\2\end{bmatrix}\right)$ . After that, find the matrix A corresponding to f, i.e. where  $f(\vec{x})=A\vec{x}$ .



**b)** Find a **diagonal** matrix D and an **orthogonal** matrix Q such that A = QD. (Not every matrix can be written in this form, but this particular A can.) Then, describe **in English** how f transforms a vector  $\vec{x}$ .



c)	Using your $A = QD$ decomposition from part <b>b</b> ), find $A^{-1}$ .								
	Hint: Recall that for orthogonal matrices, $QQ^T = Q^TQ = I$ . And, for any invertible matrices A and B, $(AB)^{-1} = B^{-1}A^{-1}$ .								
d)	Recall from Chapter 2.9 that the <b>determinant</b> of an $n \times n$ matrix $A$ , $det(A)$ , describes how much the matrix scales the "volume" of an $n$ -dimensional cube with side length 1.								
	Given the English definition of $f$ from part <b>b)</b> alone, find $det(A)$ . (Don't skip to the next page!)								

e) In general, the determinant of a 
$$3\times 3$$
 matrix  $M=\begin{bmatrix} a & b & c\\ d & e & f\\ g & h & i \end{bmatrix}$  is given by

$$\underbrace{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}_{\det(M)} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

For instance, the  $-b \begin{vmatrix} d & f \\ g & i \end{vmatrix}$  term in the determinant involves deleting row 1 and column 2 of M and taking the determinant of the remaining  $2 \times 2$  matrix.

Use this formula directly on *A* from part **a**) to verify that your intuitive answer from part **d**) is correct.

f) Find the determinant of

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

What do you notice?