EECS 245, Winter 2026

LEC 5 Vectors

This week, read Ch. 3

Agenda

- What have we learned so far, and what do we need vectors for? 2.5
- Vectors 3.1-3.2
  - Definition
  - Norm
  - Addition and scalar multiplication
  - Linear combinations

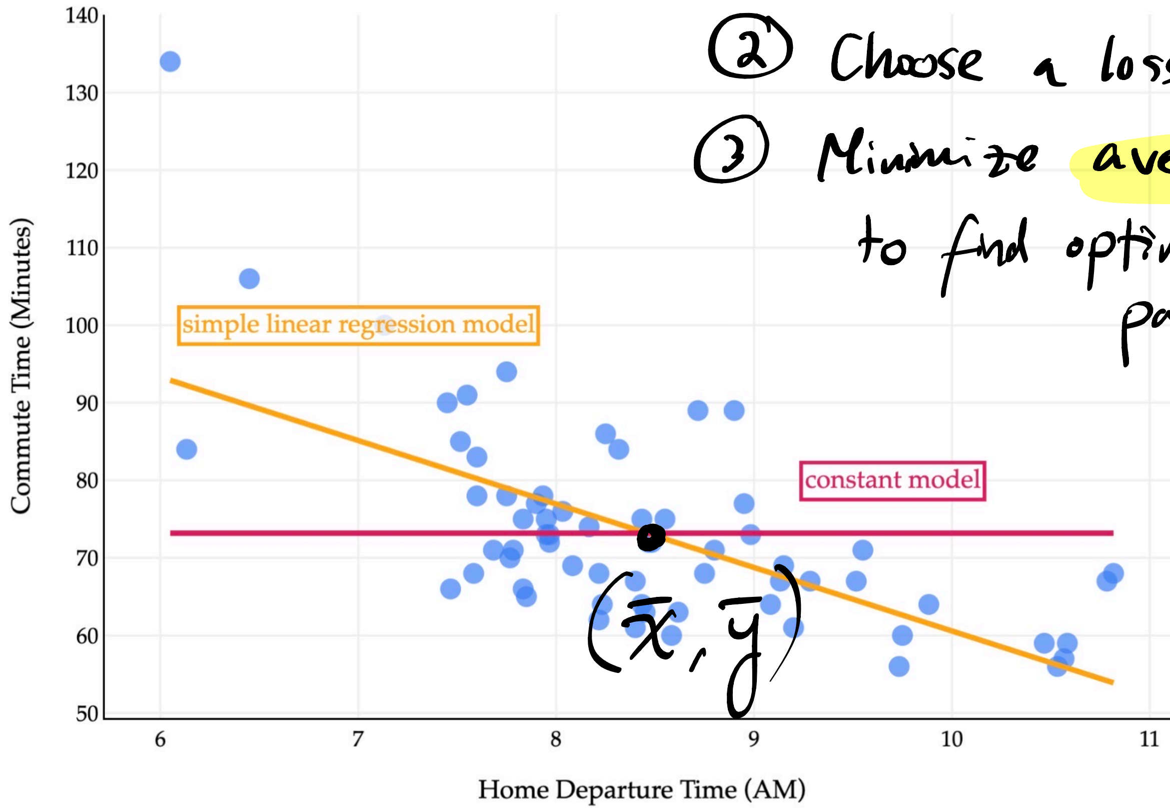
Announcements

- HW 3 due on ~~Friday~~ Monday
  - Includes a survey
  - HW 4 deadline still next Friday, so finish early
- HW 1 regrades due tomorrow
- HW 2 sol'ns up
- Will email about alt. exams tomorrow

① Choose a model

② Choose a loss function

③ Minimize average loss to find optimal parameters



	date	day	departure_hour	minutes
0	5/15/2023	Mon	10.816667	68.0
1	5/16/2023	Tue	7.750000	94.0
2	5/22/2023	Mon	8.450000	63.0
3	5/23/2023	Tue	7.133333	100.0
4	5/30/2023	Tue	9.150000	69.0

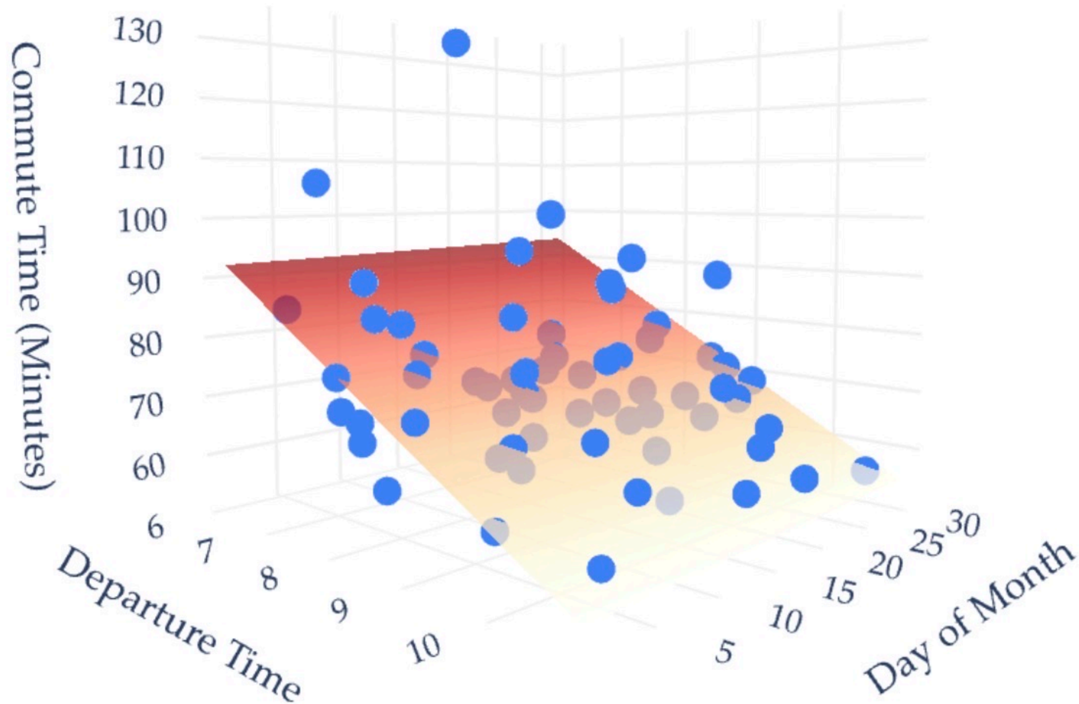
↓ day of month

"Multiple" linear regression

→ multiple features

$$h(\text{dept. hour}_i, \text{dom}_i) = w_0 + w_1(\text{dept. hour}_i) + w_2(\text{dom}_i)$$

plane in 3D!



Commute Time (Minutes)
130
120
110
100
90
80
70
60
Departure Time
6
7
8
9
10
Day of Month
5
10
15
20
25
30

$$h(\text{dept. hour}_i, \text{dom}_i) = w_0 + w_1(\text{dept. hour}_i) + w_2(\text{dom}_i)$$

$$R_{sq}(w_0, w_1, w_2) = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \left(w_0 + w_1(\text{dept. hour}_i) + w_2(\text{dom}_i)\right)\right)^2$$

→ To find w_0^*, w_1^*, w_2^*, we need to compute $\frac{\partial R}{\partial w_0}$, $\frac{\partial R}{\partial w_1}$, $\frac{\partial R}{\partial w_2}$ and solve when all are 0

→ let's find a more efficient solution!

Vectors (Ch. 3.1)

vector : an ordered list of numbers

$$\vec{v} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} \qquad \vec{w} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

3 components

"in"

$$\vec{v}, \vec{w} \in \mathbb{R}^3$$

"$\vec{v}$ and $\vec{w}$ are in R three"

$\vec{v} \in \mathbb{R}^n$

one number

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{bmatrix}$$

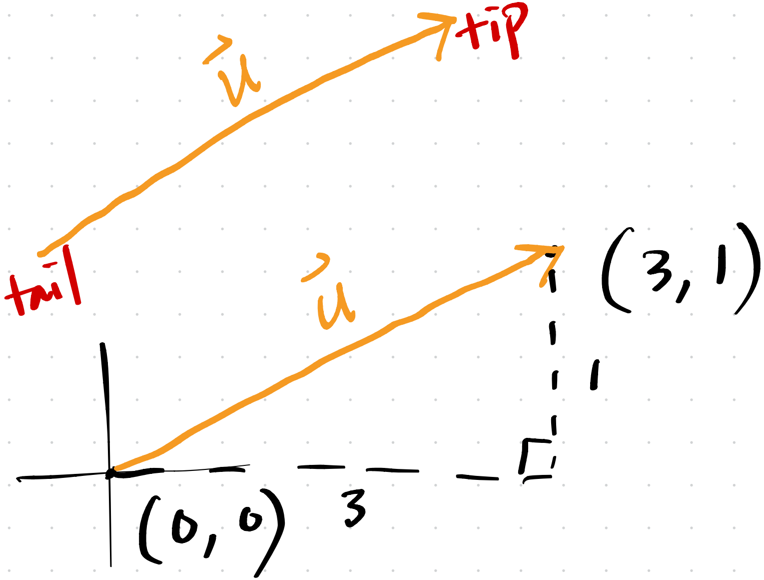
here, v_1 is a scalar

but in a different context,

$\vec{v}_1$ could refer to a vector

"Length" of a vector

$\vec{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$



$$\text{length} = \sqrt{3^2 + 1^2} = \sqrt{10} = \|\vec{u}\|$$

$\vec{v} \in \mathbb{R}^n$

called the L_2 norm, which is the default (L_1 also exists, so do others)

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2} = \sqrt{\sum_{i=1}^{n} v_i^2}$$

formal term: "norm"
means the same as "length" or "magnitude"

① Addition

$\underbrace{\vec{u}, \vec{v} \in \mathbb{R}^n}$

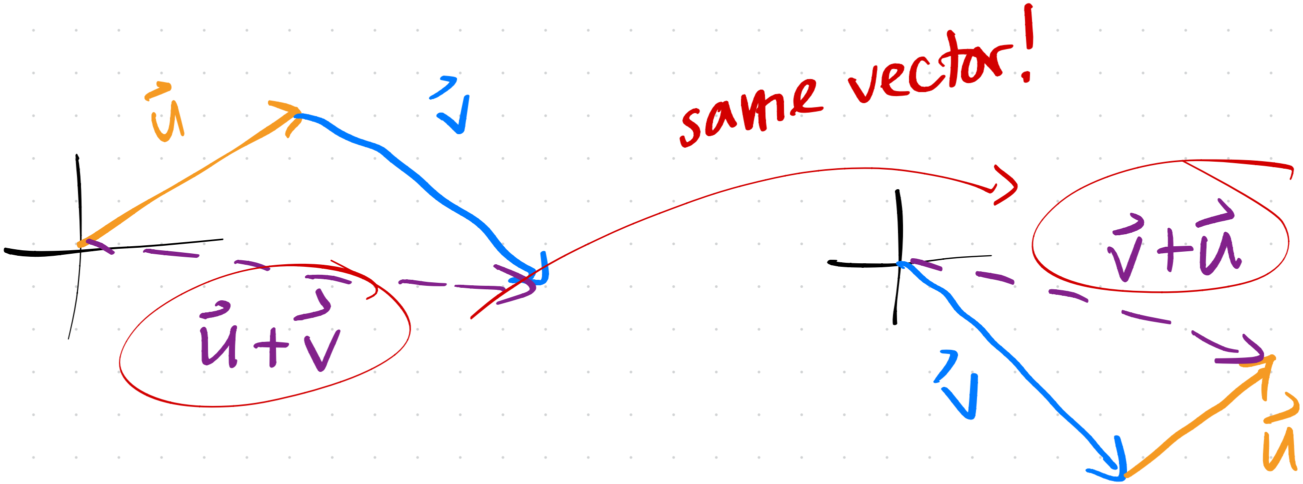
need to have same number of components

$$\vec{u} + \vec{v} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ u_3 + v_3 \\ \vdots \\ u_n + v_n \end{bmatrix} = \vec{v} + \vec{u}$$

"element-wise"

vector addition is "COMMUTATIVE"

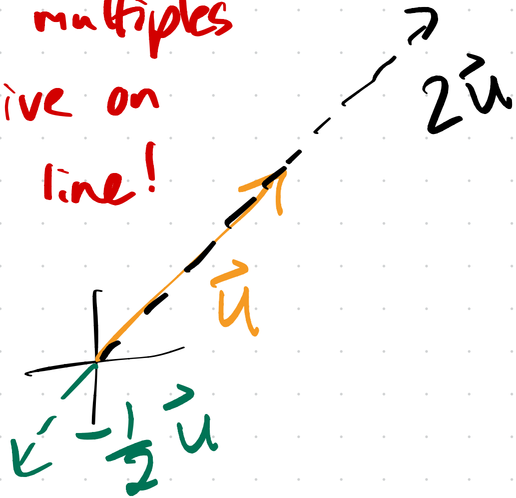
$$\vec{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} 4 \\ -6 \end{bmatrix}$$



② scalar multiplication

i.e. multiplying a vector by a number

all scalar multiples of $\vec{u}$ live on the same line!



scalar multiplication changes length!

$$c\vec{v} = \begin{bmatrix} c\,v_1 \\ c\,v_2 \\ c\,v_3 \\ \vdots \\ c\,v_n \end{bmatrix}$$

"Linear combination"

"a little bit of $\vec{u}$ plus a little bit of $\vec{v}$"

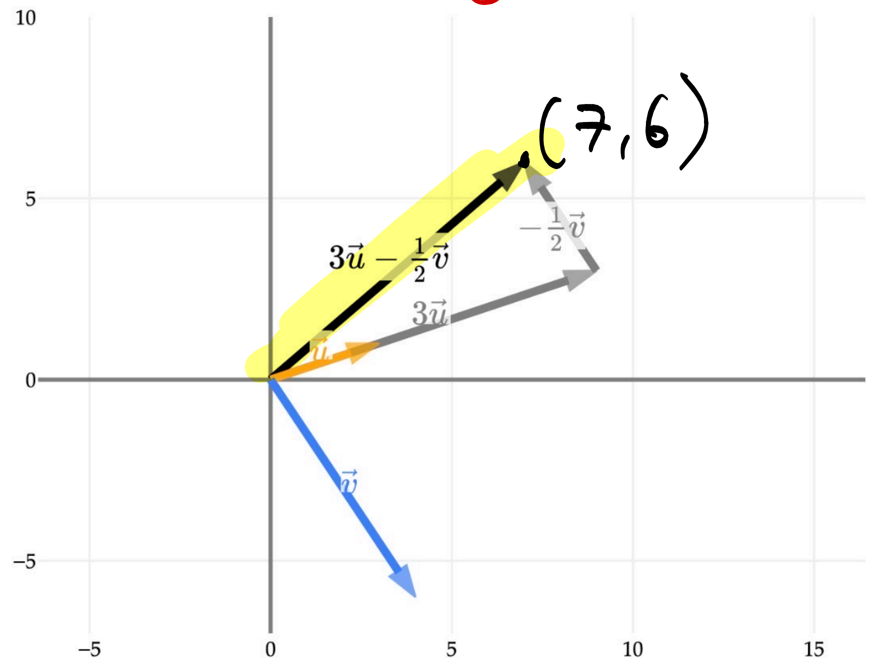
$$\vec{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 4 \\ -6 \end{bmatrix}$$

$\vec{u}, \vec{v}$ are building blocks

$$3\vec{u} - \frac{1}{2}\vec{v}$$

$$= 3\begin{bmatrix} 3 \\ 1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 4 \\ -6 \end{bmatrix}$$

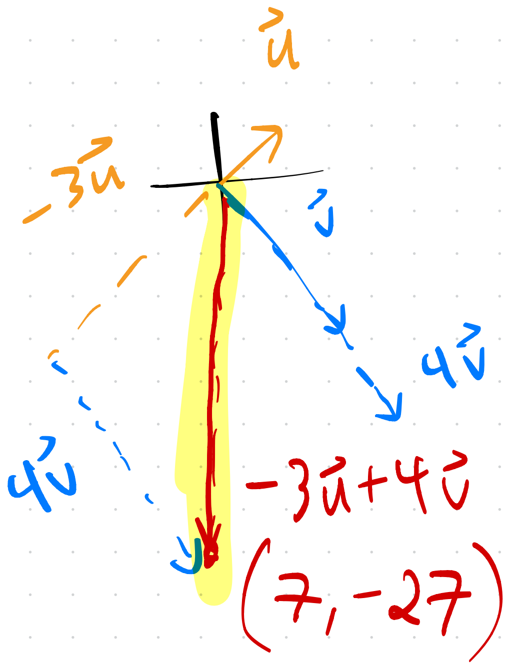
$$= \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

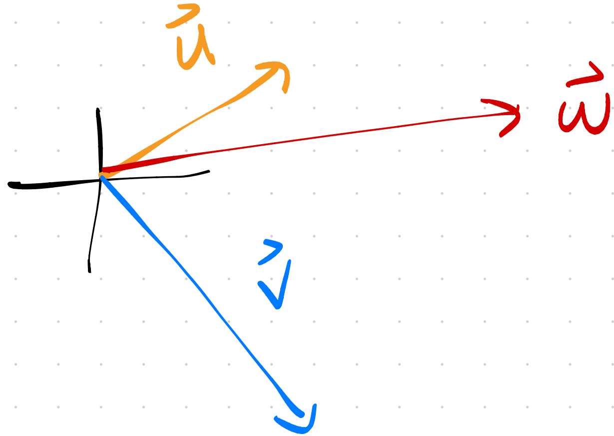


$\vec{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} 4 \\ -6 \end{bmatrix}$

$$-3\vec{u} + 4\vec{v} = \begin{bmatrix} -9+16 \\ -3-24 \end{bmatrix} = \begin{bmatrix} 7 \\ -27 \end{bmatrix}$$

$-3\vec{u} + 4\vec{v}$




$\vec{u}$
$\vec{w}$
$\vec{v}$

General definition

suppose we have d vectors,

each in $\mathbb{R}^n$,

$\vec{v}_1, \vec{v}_2, \vec{v}_3, \ldots, \vec{v}_d$

A linear combination of those vectors is any other vector that can be written as

$$a_1 \vec{v}_1 + a_2 \vec{v}_2 + \cdots + a_d \vec{v}_d$$

where $a_1, \ldots, a_d$ are scalars!

Activity

$\vec{x} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$ $\vec{y} = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$

their linear combinations fill a plane

Goal: write $\begin{bmatrix} 9 \\ -16 \\ -1 \end{bmatrix}$ as a linear combination of $\vec{x}$ and $\vec{y}$

$$c\vec{x} + d\vec{y} = \begin{bmatrix} 9 \\ -16 \\ -1 \end{bmatrix}$$

$$c\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} + d\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 9 \\ -16 \\ -1 \end{bmatrix}$$

$$3c + d = 9$$
$$-c + 4d = -16$$
$$2c + 3d = -1$$

solve: $c = 4,\ d = -3$