EECS 245 Fall 2025

Math for ML

Lecture 5: Vectors and the Dot Product

→ Read 1.5, 2.1, and 2.2!

Announcement: Check pinned posts on Ed about Homework 2! (hints/clarification)
→ I have office hours after lecture

1. Recap/wrap up simple linear regression — Ch. 1.4 (added some new activities)

2. What's next? } Ch. 1.5,

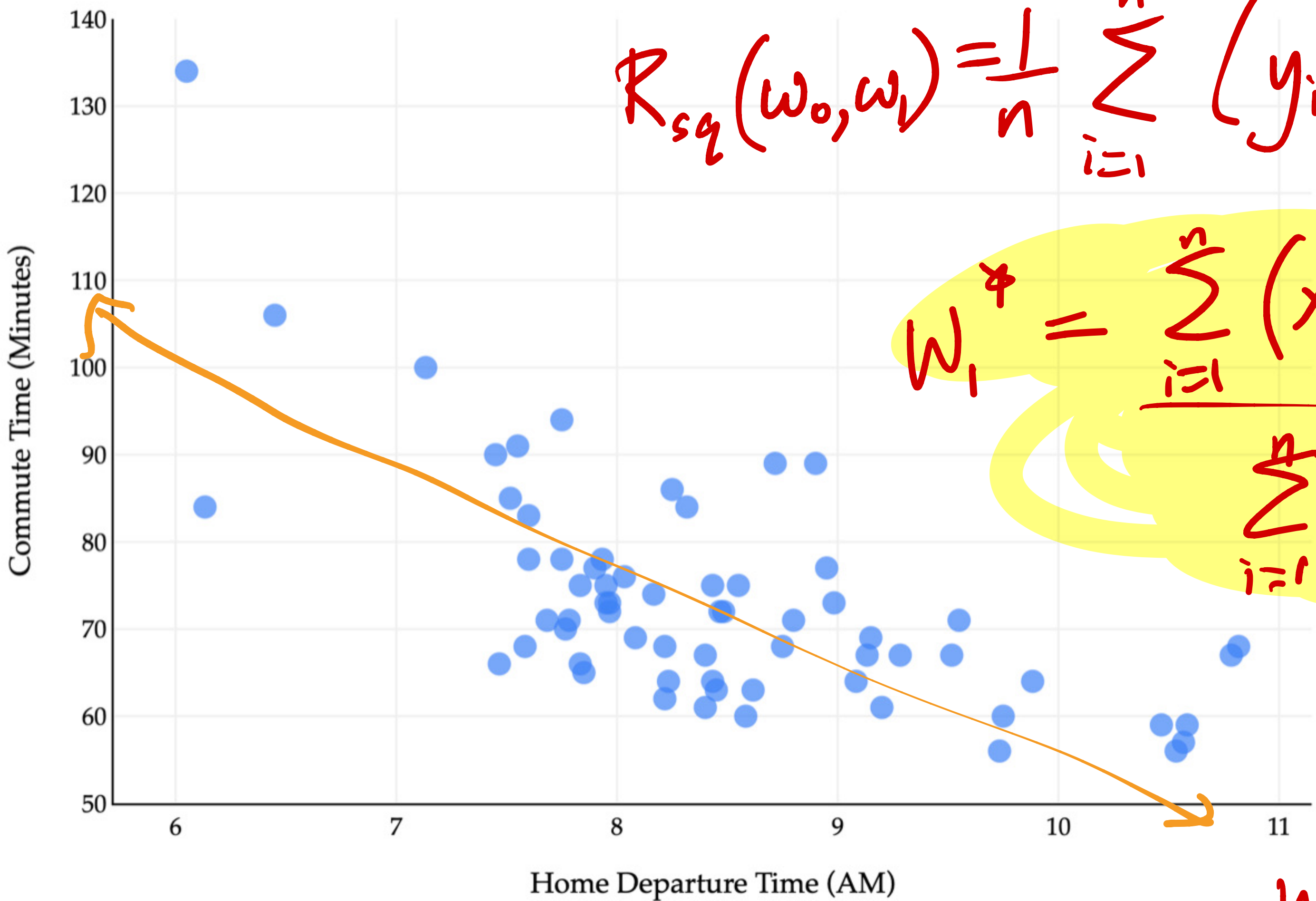
3. Vectors (norm, addition, scalar multiplication) } Ch. 2.1

4. Dot Product } Ch. 2.2

Recap

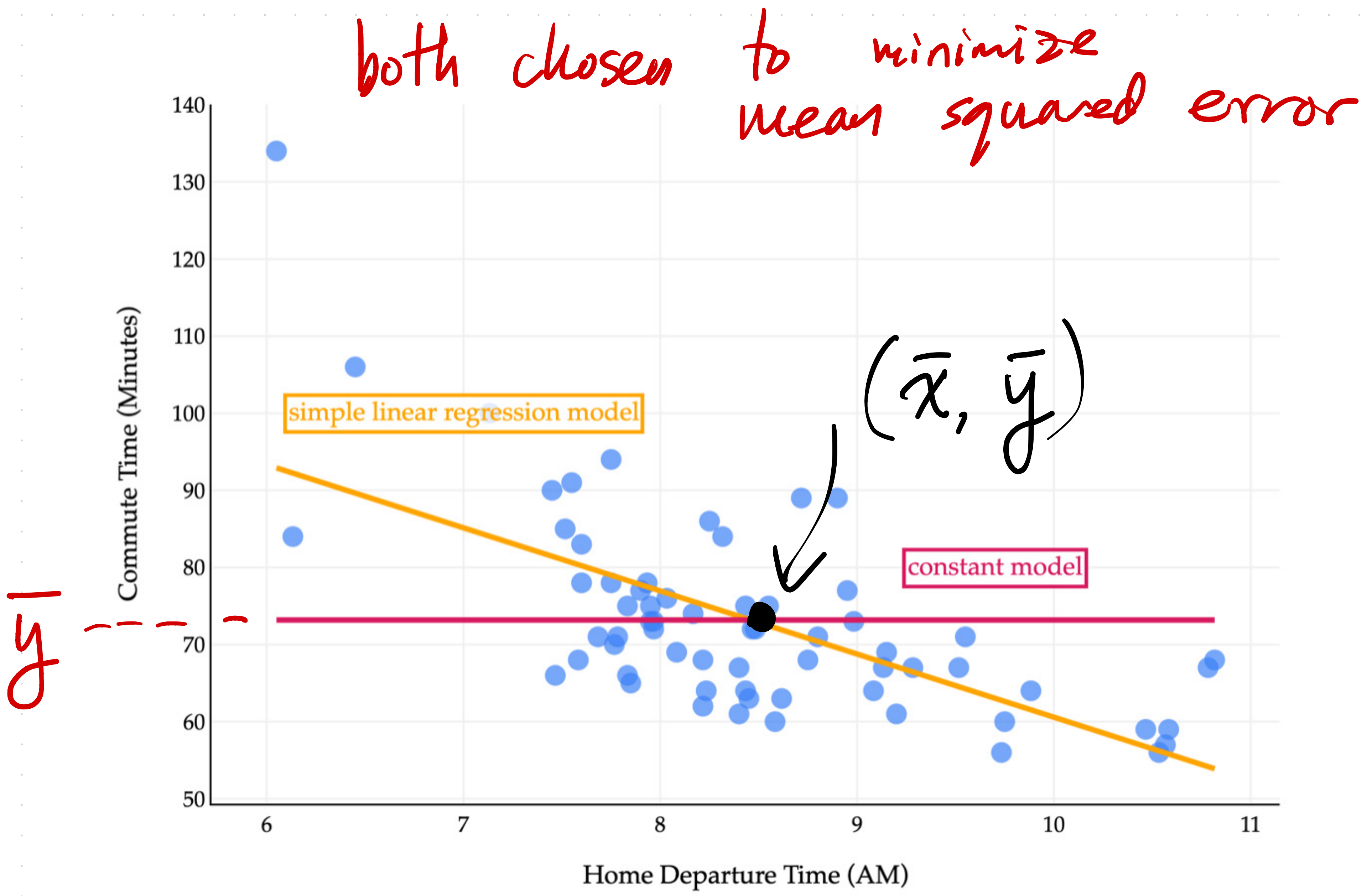
chose intercept w_0, slope w_1 that minimize mean squared error

$$R_{sq}(w_0, w_1) = \frac{1}{n} \sum_{i=1}^{n} \left(y_i - (w_0 + w_1 x_i)\right)^2$$



$$w_1^* = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} = r \frac{\sigma_y}{\sigma_x}$$

$$w_0^* = \bar{y} - w_1^* \bar{x}$$


both chosen to minimize mean squared error
$(\bar{x}, \bar{y})$
$\bar{y}$
simple linear regression model
constant model
Commute Time (Minutes)
140
130
120
110
100
90
80
70
60
50
Home Departure Time (AM)
6
7
8
9
10
11

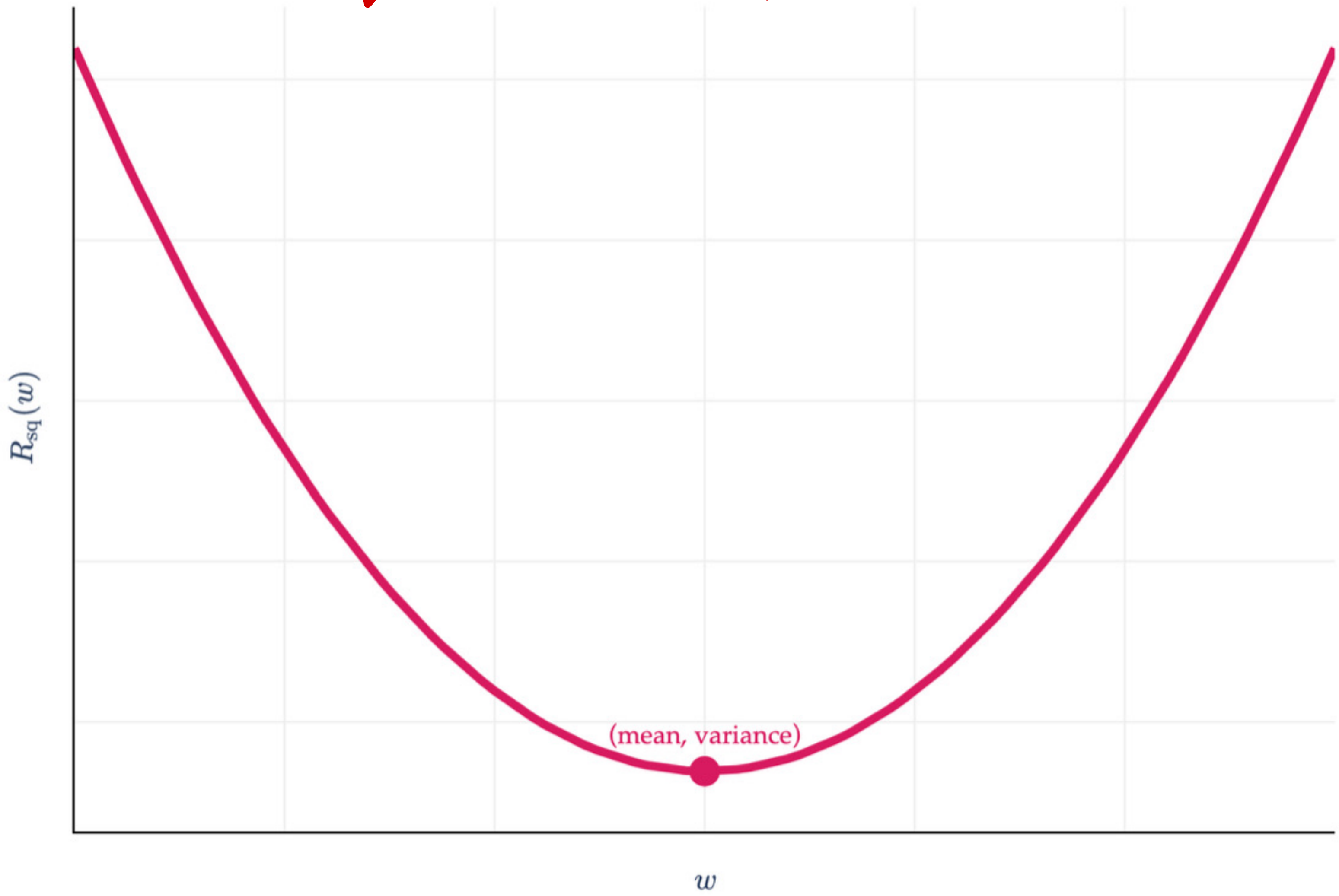
MSE of the best constant, $\bar{y}$, is $\underbrace{\sigma_y^2}_{\text{variance}}$

$$R_{sq}(w) = \frac{1}{n} \sum_{i=1}^{n} (y_i - w)^2$$

plug in $w^* = \bar{y}$

$$R_{sq}(\bar{y}) = \underbrace{\frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2}_{\text{definition of variance of } y!}$$

$$R_{sq}(w) = \frac{1}{n} \sum_{i=1}^{n} (y_i - w)^2$$



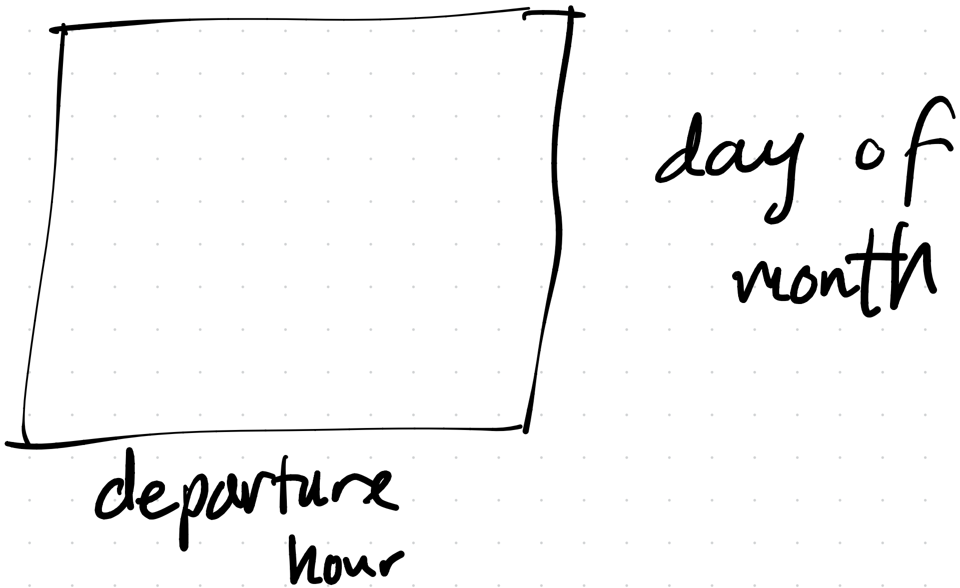
Multiple inputs

$$z = w_0 + w_1 x + w_2 y$$

$$h(dh_i, dom_i) = w_0 + w_1 \cdot \text{departure hour}_i + w_2 \cdot \text{day of month}_i$$

"Multiple linear regression" model

birds eye view



plane!

$$R(w_0, w_1, w_2)$$

$$= \frac{1}{n} \sum_{i=1}^{n} \left(y_i - \left(w_0 + w_1 \, \text{departure hour}_i + w_2 \, \text{day of month}_i \right) \right)^2$$

→ need to find $\frac{\partial R}{\partial w_0}$, $\frac{\partial R}{\partial w_1}$, $\frac{\partial R}{\partial w_2}$

and solve where all are 0

→ there's a more efficient solution: linear algebra

Linear algebra (ch. 2.1)

Vector : an ordered list of numbers

$$\vec{v} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} \quad \text{(3 components)} \qquad \vec{w} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \quad \text{(3 components)}$$

$\mathbf{v}$

$$\vec{v}, \vec{w} \in \mathbb{R}^3$$

"v and w are in R three"

$\vec{v} \in \mathbb{R}^n$

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

v_1 : scalar

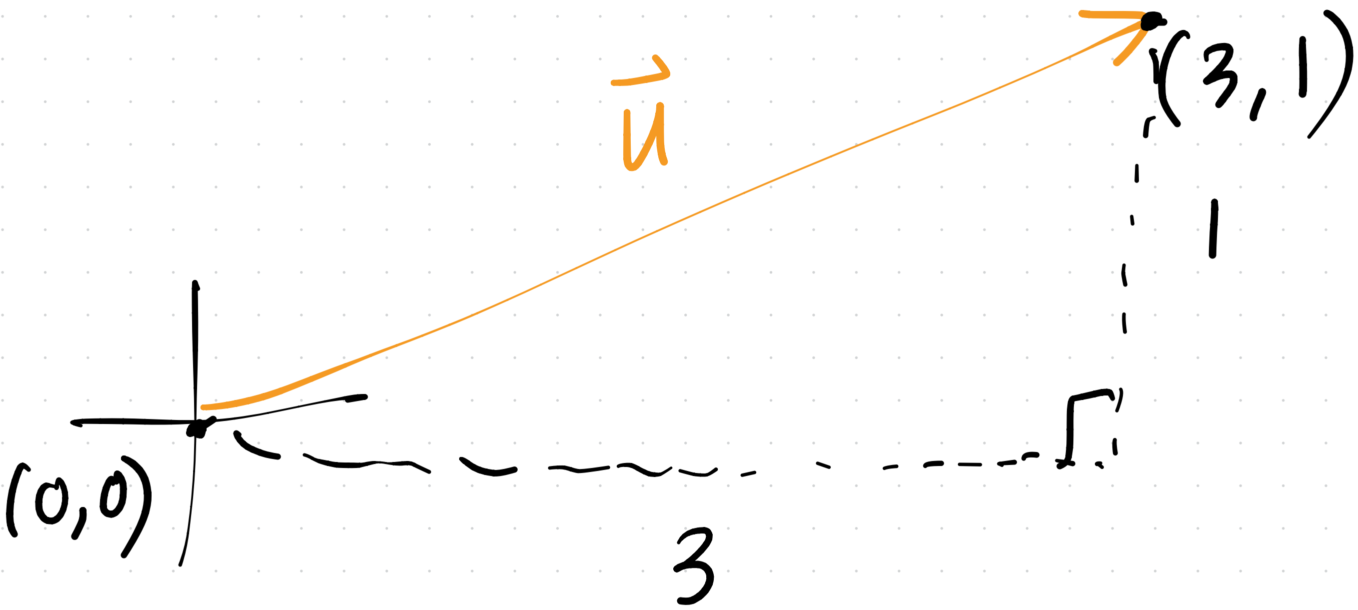
$\vec{v_1}$: vector

"column vectors"

"Length" of a vector

$\vec{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

"3 units right, 1 unit up"



$$\text{length} = \sqrt{3^2 + 1^2} = \sqrt{10} = \|\vec{u}\|$$

"norm" "length" "magnitude"

$\vec{v} \in \mathbb{R}^n$

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$$

① Addition

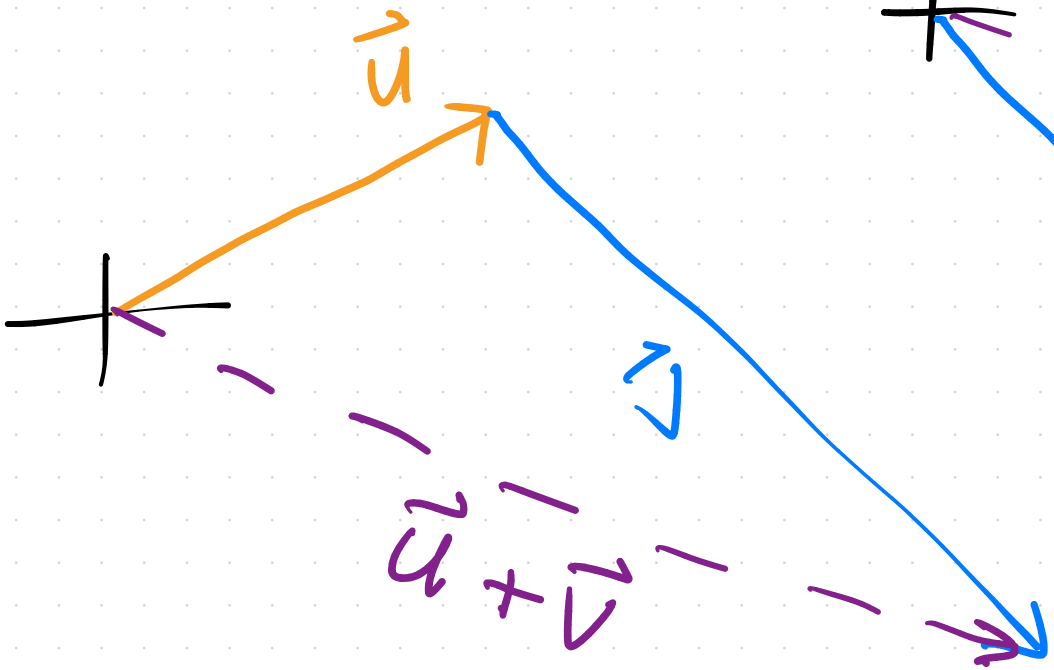
$$\vec{u} + \vec{v} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ \vdots \\ u_n + v_n \end{bmatrix}$$

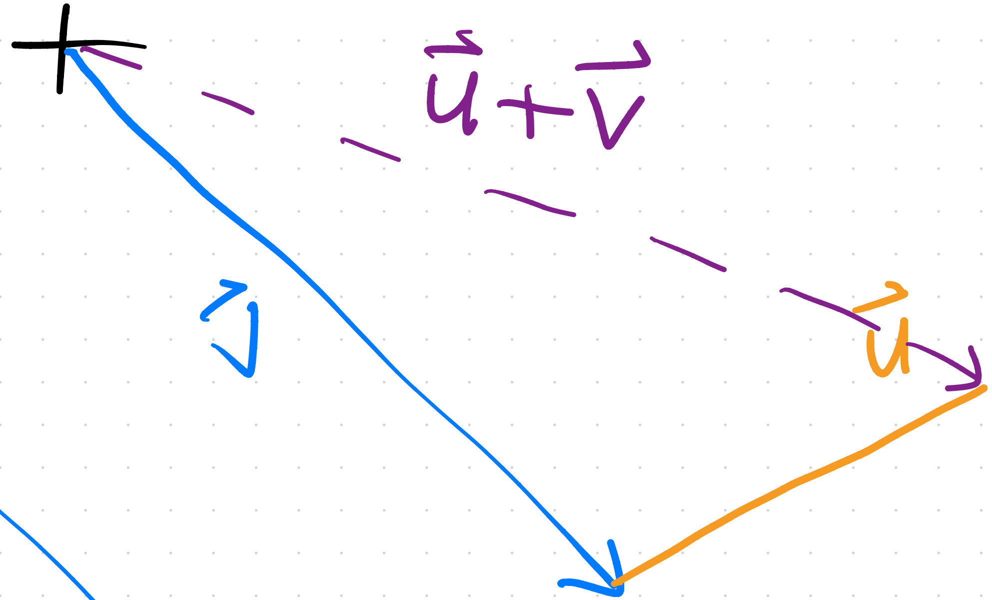
"element wise"

$$= \vec{v} + \vec{u}$$

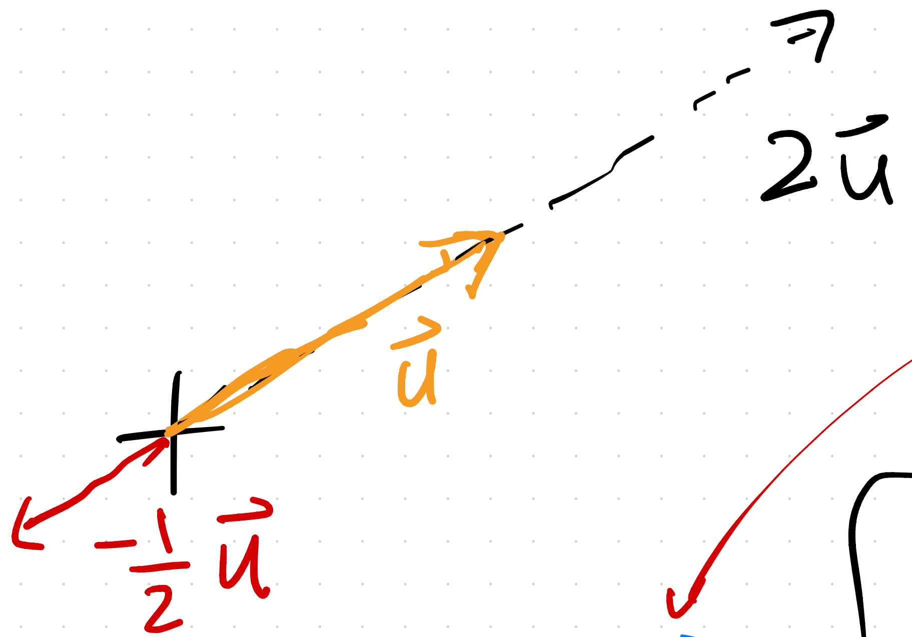
"commutative"

$$\vec{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} 4 \\ -6 \end{bmatrix}$$





② scalar multiplication



$$c\vec{v} = \begin{bmatrix} c v_1 \\ c v_2 \\ \vdots \\ c v_n \end{bmatrix}$$

$\vec{v}$: vector

c: scalar (one number)

Activity

Find $\vec{x}$:

$$3\begin{bmatrix}7\\3\\-2\end{bmatrix} + 4\vec{x} = \begin{bmatrix}9\\4\\2\end{bmatrix}$$

scalar mult.

$$\begin{bmatrix}21\\9\\-6\end{bmatrix} + 4\vec{x} = \begin{bmatrix}9\\4\\2\end{bmatrix}$$

$$4\vec{x} = \begin{bmatrix}-12\\-5\\8\end{bmatrix}$$

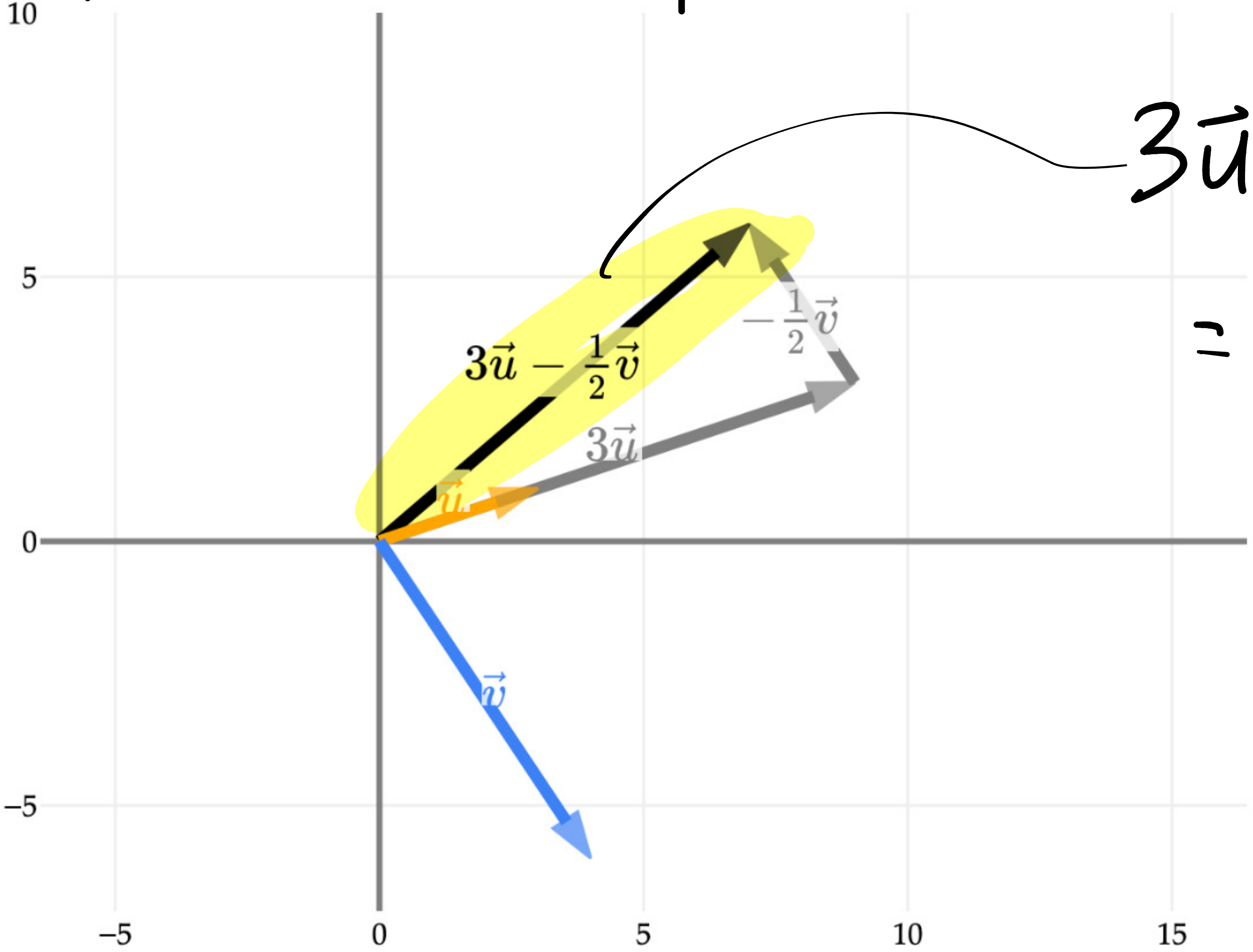
$$\vec{x} = \begin{bmatrix}-3\\-5/4\\2\end{bmatrix}$$

scalar mult.

"Linear combinations"

"a little bit of $\vec{u}$ + a little bit of $\vec{v}$"

$\vec{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $\vec{v} = \begin{bmatrix} 4 \\ -6 \end{bmatrix}$



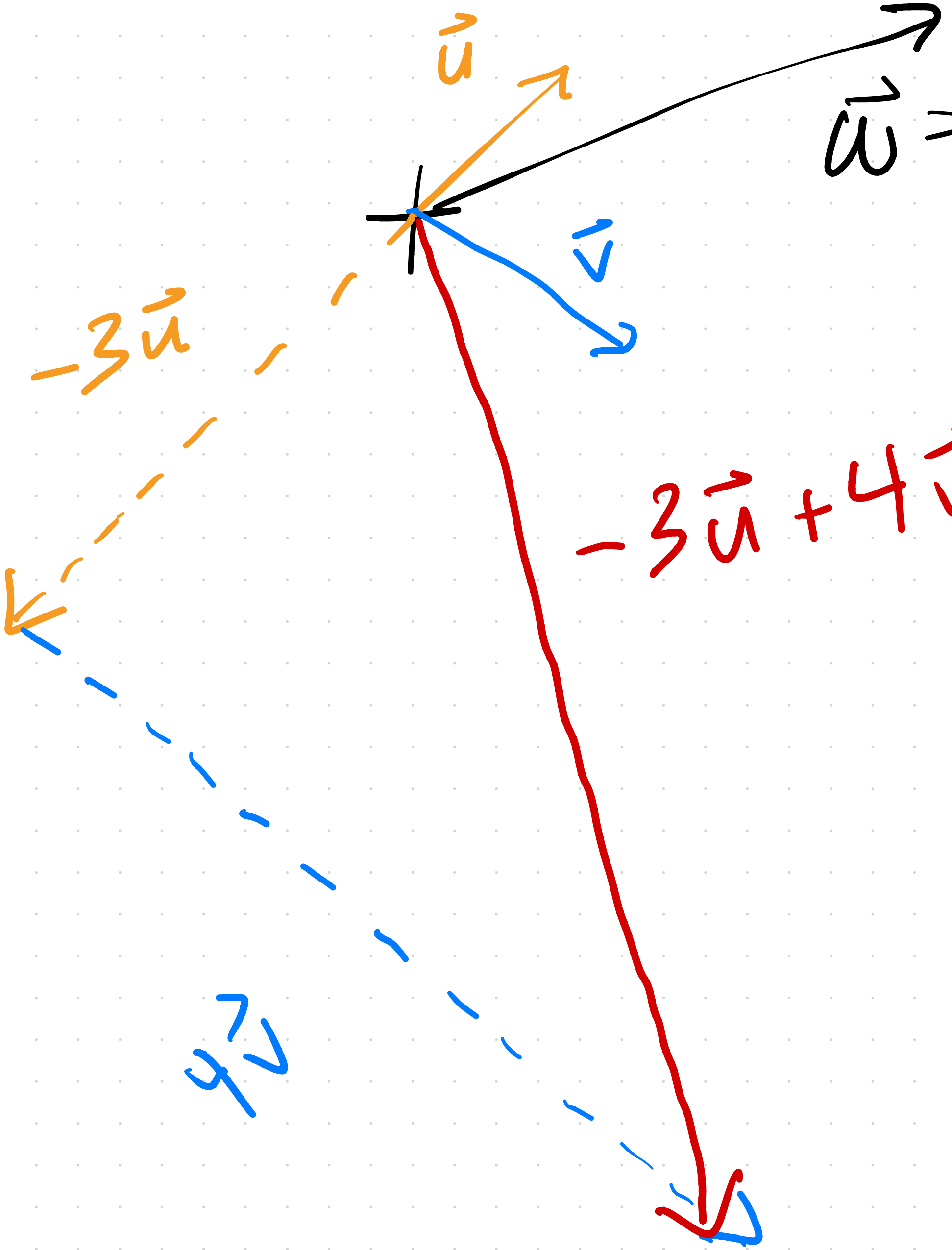
$$3\vec{u} - \frac{1}{2}\vec{v}$$

$$= 3\begin{bmatrix} 3 \\ 1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 4 \\ -6 \end{bmatrix}$$

$$= \begin{bmatrix} 9 \\ 3 \end{bmatrix} + \begin{bmatrix} -2 \\ +3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

$$-3\vec{u}+4\vec{v} = \begin{bmatrix} -9 \\ -3 \end{bmatrix} + \begin{bmatrix} 16 \\ -24 \end{bmatrix} = \begin{bmatrix} 7 \\ -27 \end{bmatrix}$$



$\vec{w} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

can we make $\vec{w}$ using $\vec{a}$, $\vec{v}$?

$\vec{v} = \begin{bmatrix} 4 \\ -6 \end{bmatrix}$

$\vec{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$-3\vec{u}+4\vec{v}$

$\begin{bmatrix} 7 \\ -27 \end{bmatrix}$ is a linear combination of

$\vec{u}$ and $\vec{v}$!

$-3\vec{u} + 4\vec{v}$

<u>General</u> $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_d \in \mathbb{R}^n$ d vectors, all are n-dimensional

A linear combination of those vectors is

$$a_1 \vec{v}_1 + a_2 \vec{v}_2 + \ldots + a_d \vec{v}_d$$

$a_1, \ldots, a_d$ scalars