EECS 245, Winter 2026

LEC 9 Vector Spaces and Subspaces

→ Read: Ch. 4.3

Agenda

→ Ch. 4 is the last chapter in scope for MT1!

- Recap: linear independence
- Vector spaces and subspaces

formalizing an idea we saw last class

- Basis and dimension

→ Wednesday's class will mostly be review of this content

Announcements

- HW 4 due today
- HW 5 due Friday
- Practice MT1 on Friday in 1014 DOW:
  - 2:30-4:30: take Mock MT1
  - 4:30-5:30: take it up (recorded)

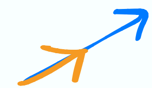
Midterm 1

- Monday, February 16th, 7-9 PM in 1670 BBB
- Content: Lectures 1-10, Chapters 1-4
  HW 1-5
  Lab 1-6
- Mix of MC, SA, short answer, and proof

  see last sem's exams for formatting details
- Can bring one 8.5×11" double-sided handwritten notes sheet (no iPad!)

Chapter 4.2, Activity 2

To recap what we've covered in this section, answer the following questions.

1. Can any three vectors in $\mathbb{R}^2$ be linearly independent? no!
2. **Must** any two vectors in $\mathbb{R}^2$ be linearly independent?  no!
3. If two vectors in $\mathbb{R}^3$ are linearly independent, what do they span? plane
4. If three vectors in $\mathbb{R}^3$ are linearly independent, what do they span? all of $\mathbb{R}^3$!
5. Given d vectors in $\mathbb{R}^n$, what must be true about d and n for it to be possible for the vectors to be linearly independent?

   \# vectors $\leq$ \# dimensions

   $d \leq n$

set of vectors

$\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_d$ are linearly independent if

① none of them can be written as
a linear combination of the others

$\updownarrow$ equivalent statements!

② the only way to create $\vec{0}$ as a lin. comb. of them is
$a_1\vec{v}_1 + a_2\vec{v}_2 + \cdots + a_d\vec{v}_d$ where $a_1 = a_2 = \cdots = 0$

→ otherwise, the set is linearly dependent

4.2 Activity 3

(1) 8 vectors in $\mathbb{R}^{17}$

- could they be linearly independent? yes, they could
- could they span all of $\mathbb{R}^{17}$? no!

(2) 17 vectors in $\mathbb{R}^8$

- could they be linearly independent? no!
- could they span all of $\mathbb{R}^8$? yes, they could

Chapter 4.3

$\mathbb{R}^n$ "Euclidean vectors"

Vector space is a set of objects, V, (called vectors) that support 2 operations:

① Addition $\vec{u}, \vec{v} \in V \Rightarrow \vec{u}+\vec{v} \in V$

② Scalar multiplication $\vec{v} \in V,\ c \in \mathbb{R} \Rightarrow c\vec{v} \in V$

$\Rightarrow$ $\vec{u}, \vec{v} \in V,\ c, d \in \mathbb{R} \Rightarrow c\vec{u}+d\vec{v} \in V$

e.g. $V =$ set of polynomials with degree ≤ 3

$u(x) = x^3 + 3x^2 - 6$

$v(x) = x^2 + 15$

need $\leq$

$$\begin{array}{r} 2x^3 + 3x^2 \\ -2x^3 + 3x^2 \\ \hline + \end{array}$$

$\Rightarrow$ deg 2 poly!

Subspace

S is a subset of V

A subspace S of a vector space V is a set where

① $\vec{u}, \vec{v} \in S \Rightarrow \vec{u} + \vec{v} \in S$

② $c \in \mathbb{R}, \vec{v} \in S \Rightarrow c\vec{v} \in S$

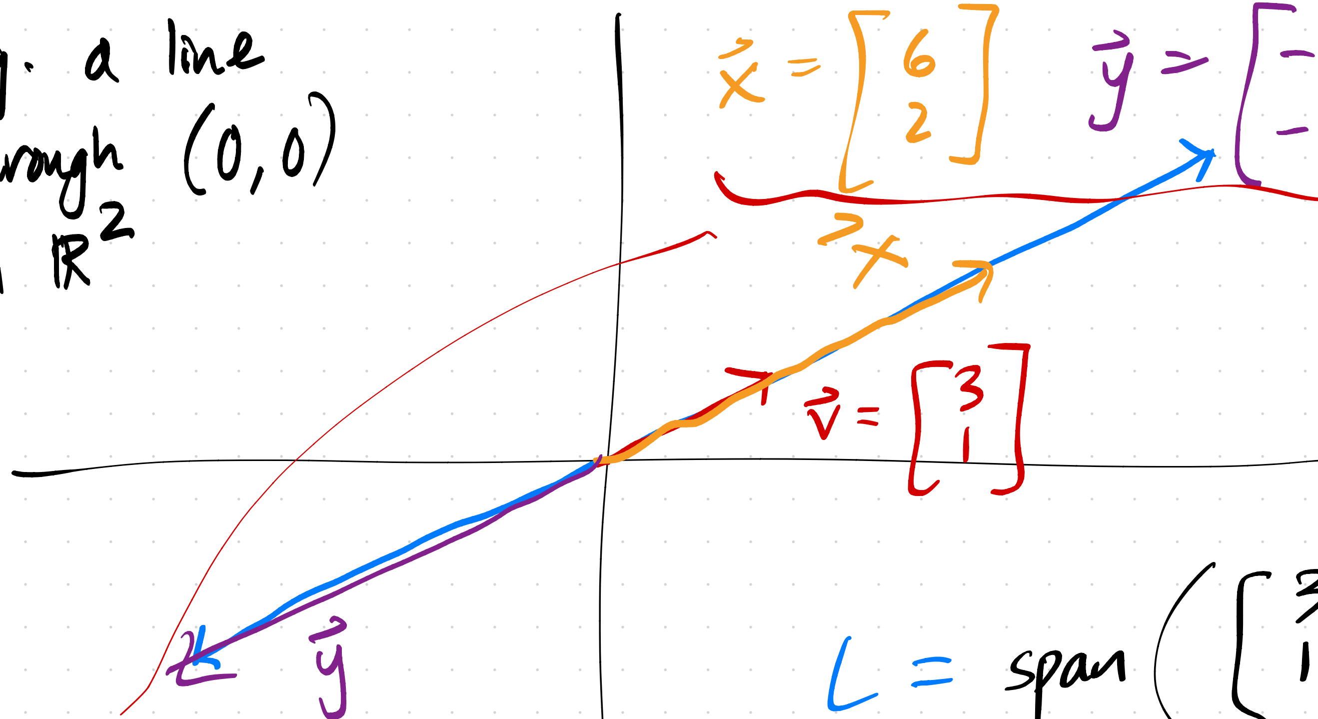
③ $\vec{0} \in S$

a subspace is a vector space!

subspace is "closed" under ⊕, scalar ⊗

e.g. a line through $(0,0)$ in $\mathbb{R}^2$

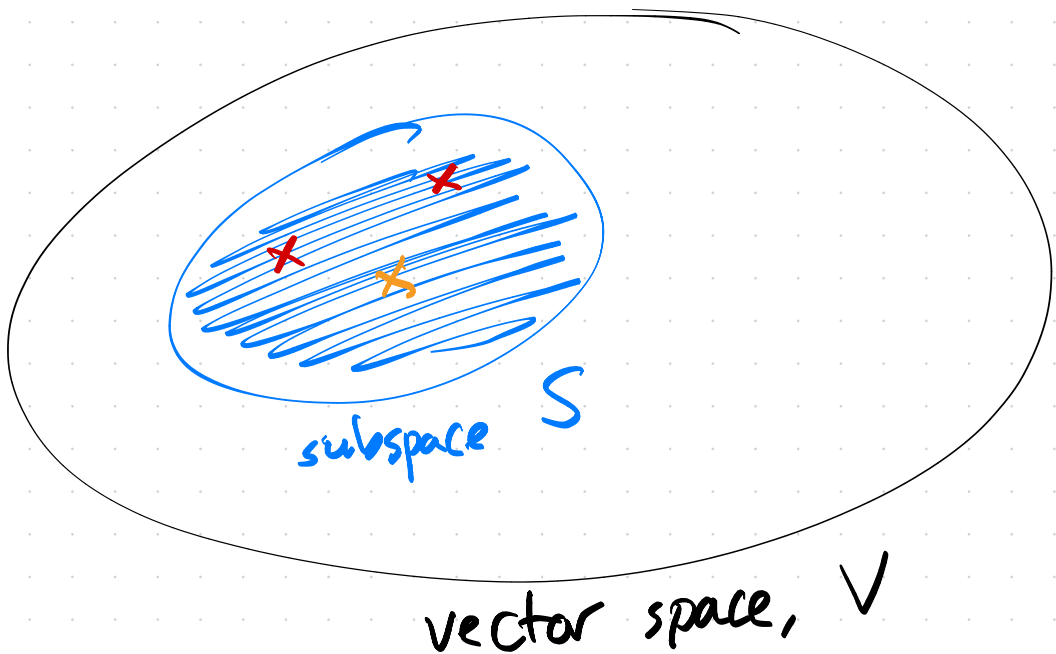
$\vec{x} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$ $\quad \vec{y} = \begin{bmatrix} -9 \\ -3 \end{bmatrix}$

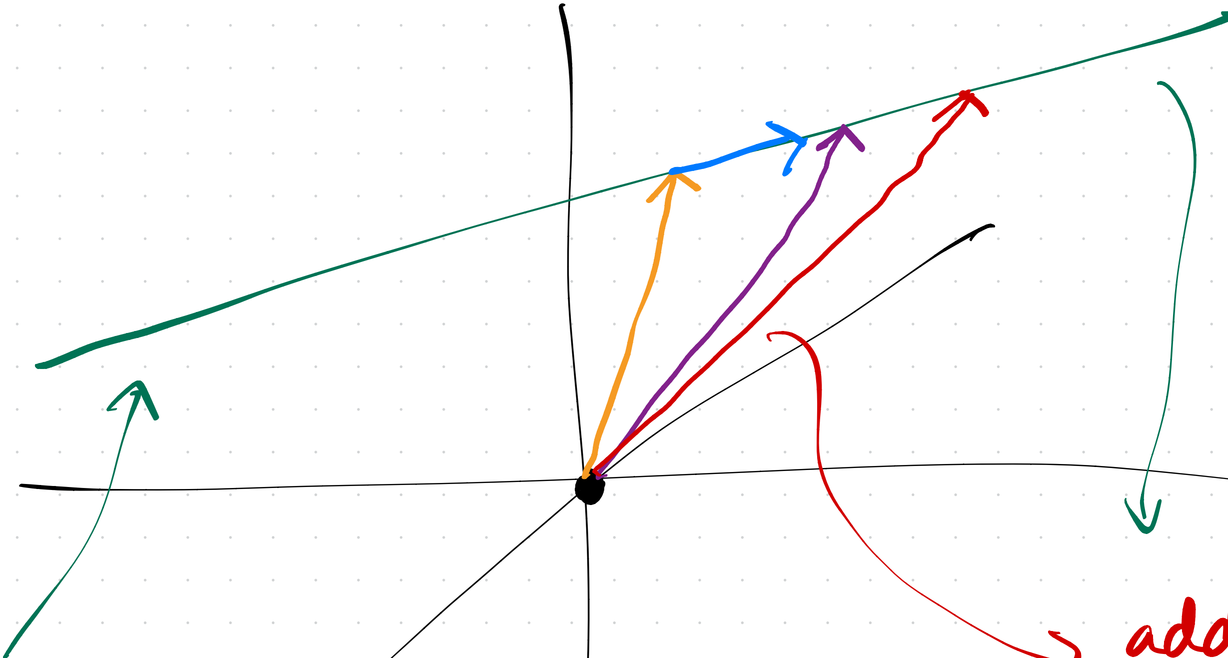


$\vec{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$$L = \text{span}\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix} \right) = \underbrace{\left\{ a \begin{bmatrix} 3 \\ 1 \end{bmatrix} \,\middle|\, a \in \mathbb{R} \right\}}_{\text{subspace of } \mathbb{R}^2}$$

any linear combination of vectors on the line will still be on the line


subspace S
vector space, V

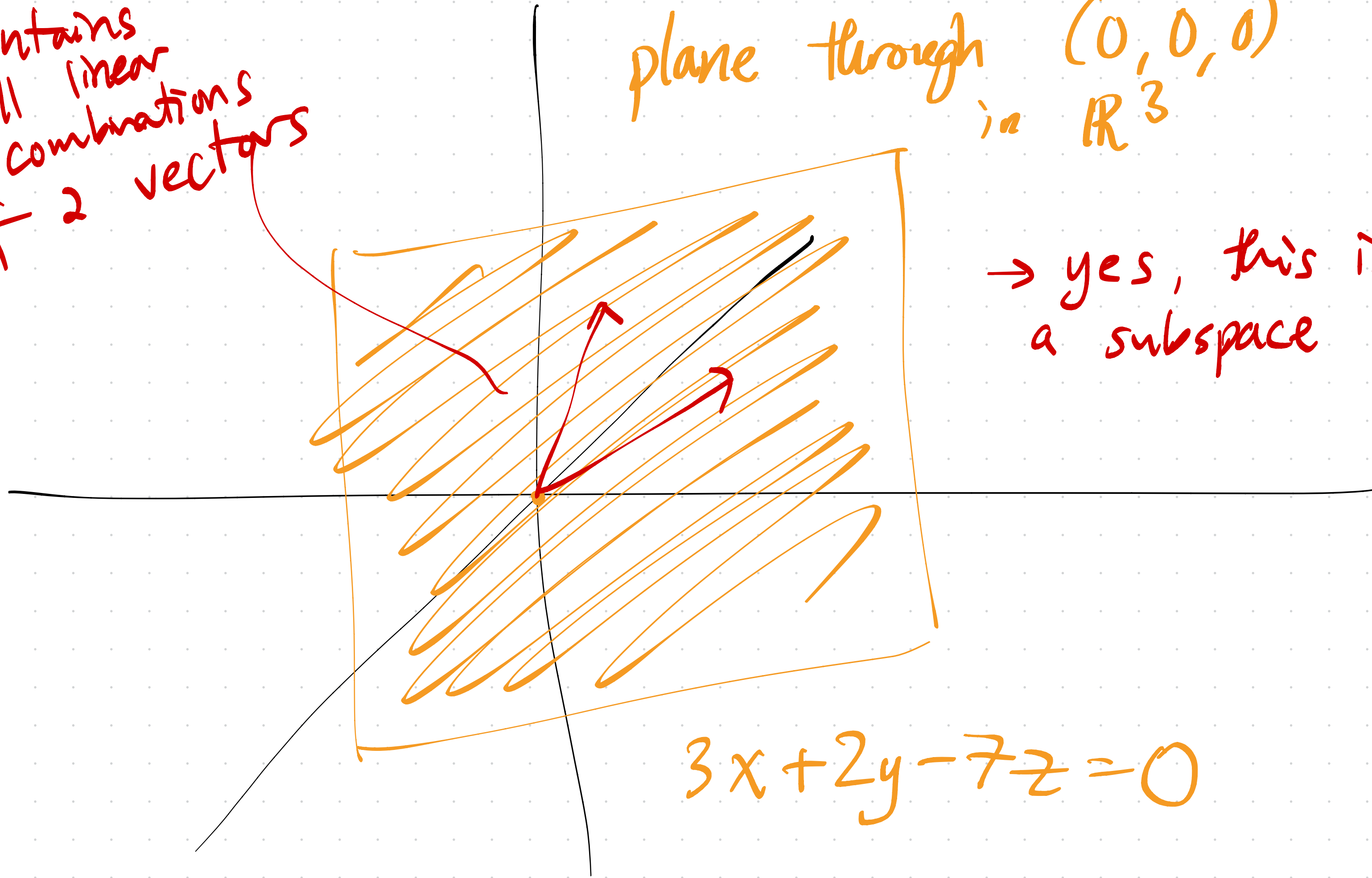


not a subspace, since it doesn't contain $\vec{0}$!

add the purple and red vectors, result is not on the line!

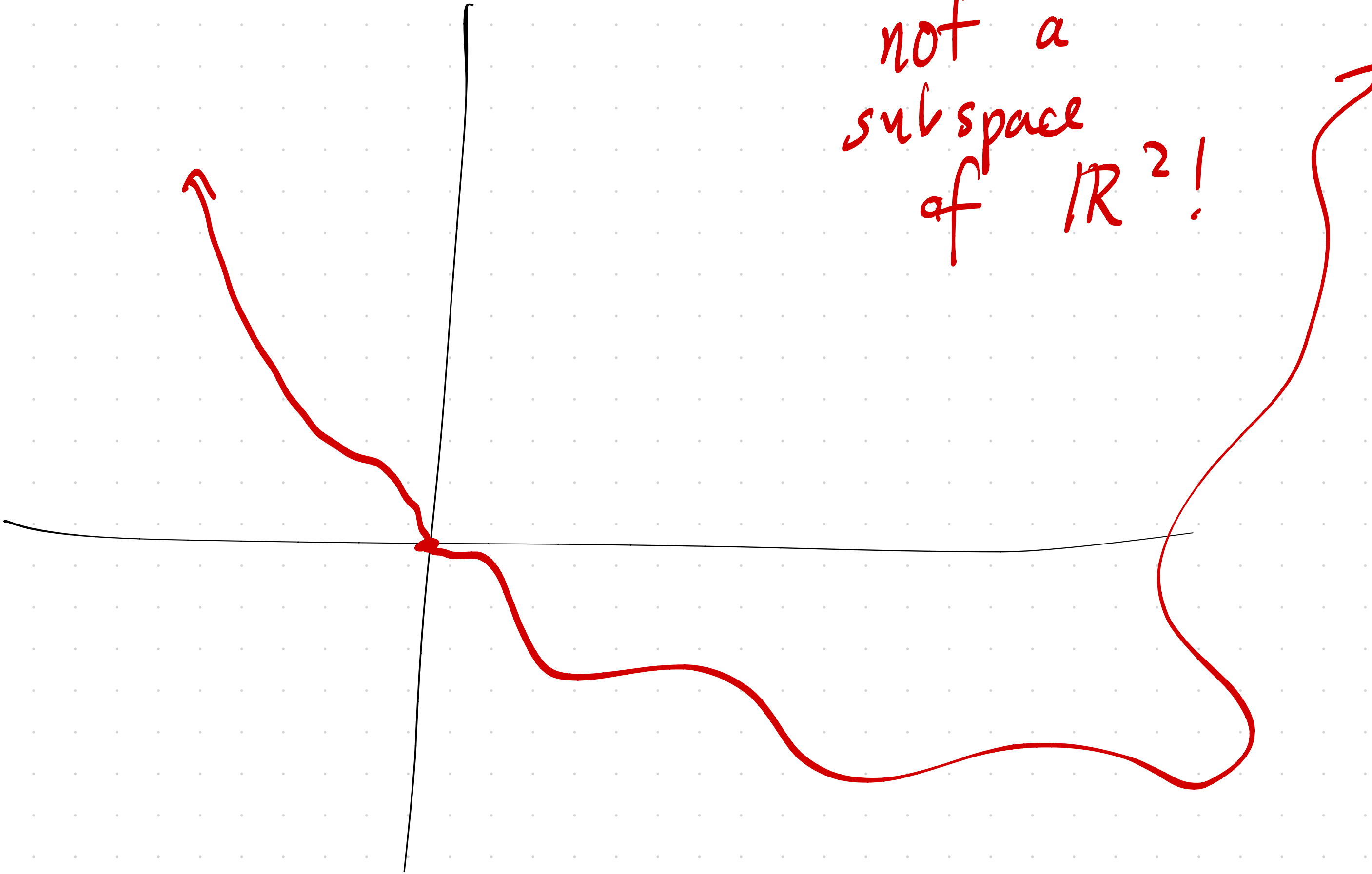
plane through $(0,0,0)$ in $\mathbb{R}^3$

contains all linear combinations of 2 vectors



→ yes, this is a subspace

$3x+2y-7z=0$



not a subspace of $\mathbb{R}^2$!

$\text{span}\left(\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_d\}\right)$

is a

subspace !

AND

all subspaces are spanned by some collection of vectors

~~ex.~~ the set of vectors in $\mathbb{R}^5$ where the second and fourth components are equal (within each vector)

① subspace?

② if so, find vectors that span it

① yes it's a subspace! $\vec{0} \in S$ and closed under linear combinations

$$\vec{u} = \begin{bmatrix} a \\ b \\ c \\ b \\ d \end{bmatrix} \quad \vec{v} = \begin{bmatrix} e \\ f \\ g \\ f \\ h \end{bmatrix}$$

$$\alpha\vec{u} + \beta\vec{v} = \begin{bmatrix} \alpha a + \beta e \\ \alpha b + \beta f \\ \alpha c + \beta g \\ \alpha b + \beta f \end{bmatrix}$$

→ still equal!

② find vectors that span this subspace

$$\begin{bmatrix} a \\ b \\ c \\ b \\ d \end{bmatrix}$$

⇗ "basis" for the subspace

$\dim(S) = 4$

$$\left\{ \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 17 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 11 \\ 0 \\ 11 \\ 0 \end{bmatrix} \right\}$$

e.x. set of vectors in $\mathbb{R}^3$ where the first component is 1

$$\vec{u} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \Rightarrow \vec{u}+\vec{v} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

not in set!

so, this is not a subspace