EECS 245, Winter 2026

LEC 10 Basis and Dimension

→ Read: Ch. 4.3

Agenda

all in scope

→ Review subspaces

→ Basis and dimension

→ If time: orthogonal complements

Announcements

- Midterm 1 Monday 7-9PM!
- HW 5 due Friday
- Grade Report updated
- Mock exam on Friday, 1014 DOW 2:30-5:30 PM
- Lecture on Monday will be office hours (in lec room)
- See videos for FA25 Midterm 1

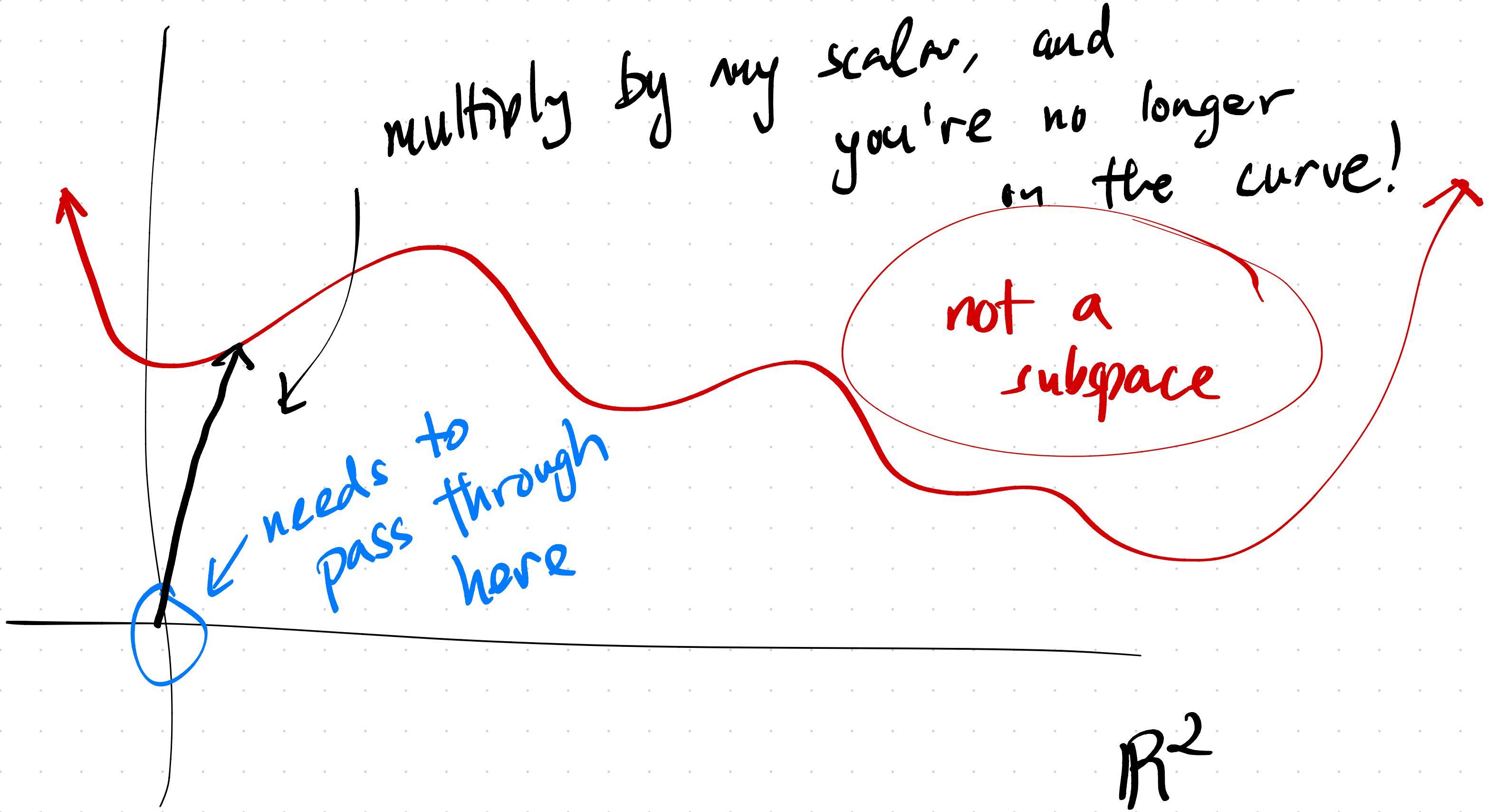
Two big ideas

① what makes a subset of $\mathbb{R}^n$ a subspace?

② span $\Longleftrightarrow$ subspace

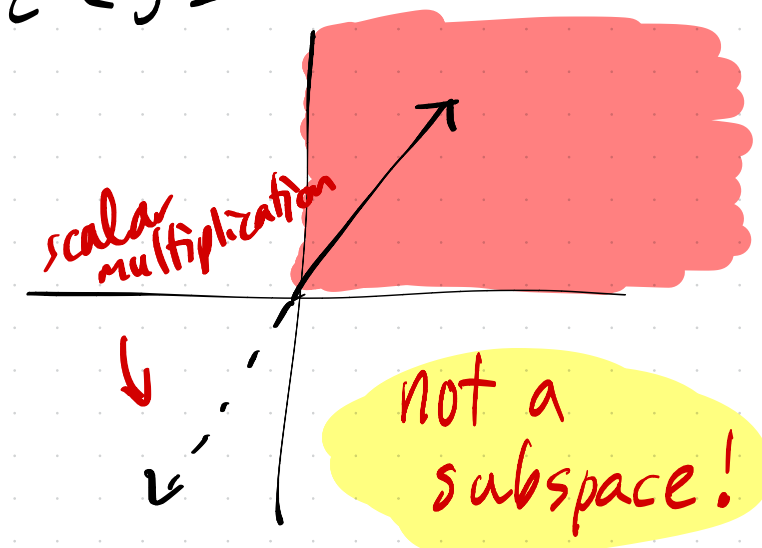
given $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_d$, describe the subspace they span

given a description of a subspace S, find vectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_d$ such that $\text{span}(\{\vec{v}_1, \dots, \vec{v}_d\}) = S$


multiply by my scalar, and you're no longer on the curve!
not a subspace
needs to pass through here
$\mathbb{R}^2$

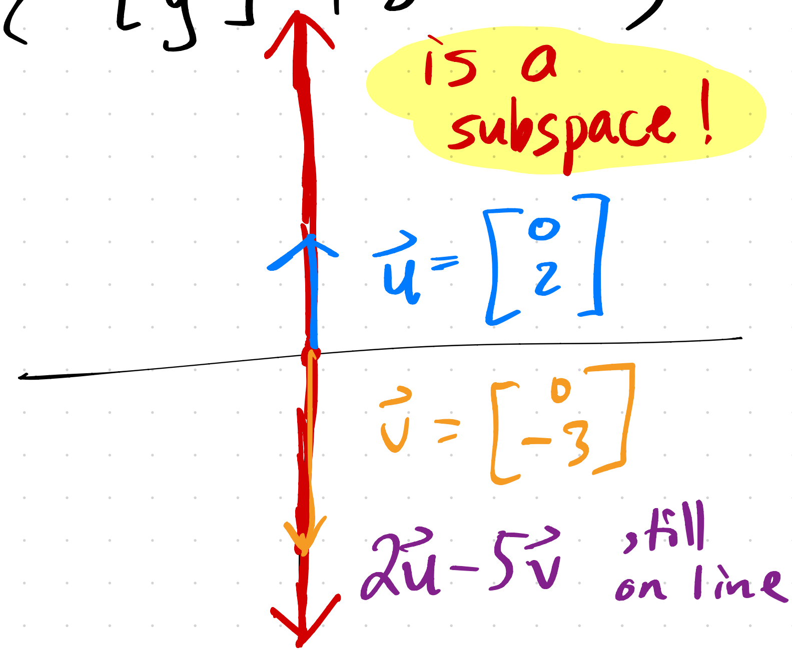
set of all vectors in $\mathbb{R}^2$ where BOTH components are ≥ 0

$$\left\{ \begin{bmatrix} x \\ y \end{bmatrix} \;\middle|\; x \geq 0,\, y \geq 0 \right\}$$



set of all vectors in $\mathbb{R}^2$ whose first component is 0

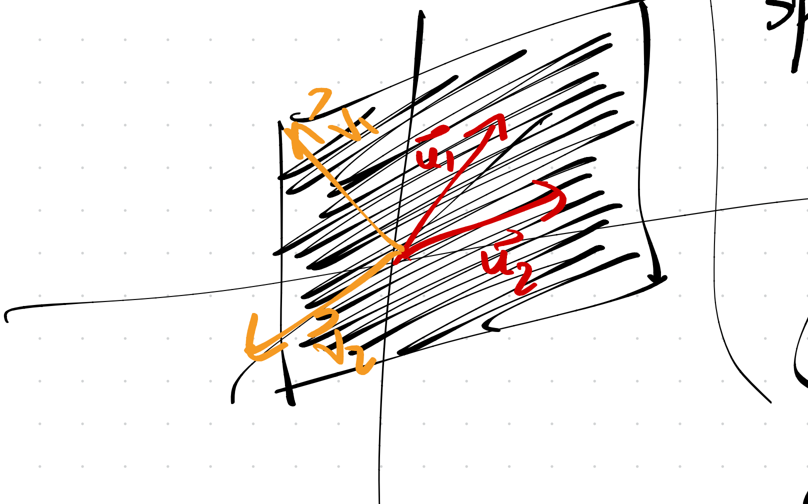
$$\left\{ \begin{bmatrix} 0 \\ y \end{bmatrix} \;\middle|\; y \in \mathbb{R} \right\}$$



$$x+y+z=0$$

plane in $\mathbb{R}^3$

this **is a subspace!**



$$\text{span}\left(\left\{\underset{\vec{u}_1}{\begin{bmatrix}1\\-1\\0\end{bmatrix}}, \underset{\vec{u}_2}{\begin{bmatrix}0\\-1\\1\end{bmatrix}}\right\}\right) \quad \text{basis}$$

$$=$$

basis

$$\text{span}\left(\left\{\underset{\vec{v}_1}{\begin{bmatrix}2\\1\\-3\end{bmatrix}}, \underset{\vec{v}_2}{\begin{bmatrix}99\\-98\\-1\end{bmatrix}}\right\}\right)$$

$$\left\{\begin{bmatrix}x\\y\\z\end{bmatrix} \,\middle|\, x+y+z=0\right\}$$

same thing

big idea:

- the span of a set of vectors is a subspace
- all subspaces can be written as the span of some set of vectors

Basis

bases : plural of basis

A basis for a subspace S is a set of vectors that:

- are linearly independent, and
- span all of S

basis is a minimal set of "building blocks" (spanning vectors)

→ every subspace has infinitely many bases!

Dimension

$\dim(S)$

is the number of vectors in any basis for S

$$2x - 3y + 4z = 0$$

1. 2 possible bases
2. dimension

$$\dim(S) = 2$$

- $\left\{ \begin{bmatrix} -7 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 6 \\ 6 \end{bmatrix} \right\}$

one of infinitely many bases

- $\left\{ \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix} \right\}$

another possibility

Aside: Parametric equation for a plane

$$P: \quad s\begin{bmatrix}-7\\2\\5\end{bmatrix} + t\begin{bmatrix}-3\\6\\6\end{bmatrix}, \quad s,t \in \mathbb{R}$$

e.g. a 3d subspace in $\mathbb{R}^6$

$$s\begin{bmatrix}1\\2\\3\\4\\5\\6\end{bmatrix} + t\begin{bmatrix}0\\0\\1\\0\\0\\1\end{bmatrix} + u\begin{bmatrix}3\\4\\5\\-17\\32\\\pi^2\end{bmatrix}, \quad s,t,u \in \mathbb{R}$$

Aside: "Standard" basis for $\mathbb{R}^n$

e.g. $\mathbb{R}^2$

$$\underbrace{\left\{ \begin{bmatrix} \pi \\ 7 \end{bmatrix}, \begin{bmatrix} -2 \\ e \end{bmatrix} \right\}}_{\text{a basis}} \qquad \underbrace{\left\{ \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \end{bmatrix} \right\}}_{\text{orthogonal basis}}$$

$$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$$ standard basis

$$\begin{bmatrix} 7 \\ -3 \end{bmatrix} = 7 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (-3) \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Why do we care about linear independence?

→ when vectors are linearly independent, any linear combination of them can only be written in one way!

e.g. $\left\{ \underset{\vec{u}}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}, \underset{\vec{v}}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}, \underset{\vec{w}}{\begin{bmatrix} 3 \\ 4 \end{bmatrix}} \right\}$

not a basis, but they span all of $\mathbb{R}^2$!

key: if vectors are not LI, any linear combination of them can be expressed infinitely many ways → bad!

$\vec{w} = 3\vec{u} + 4\vec{v}$

$\vec{0} = 3\vec{u} + 4\vec{v} - \vec{w}$

$\vec{x} = 2\vec{u} + 3\vec{v} + \vec{w}$

$(-1.72)\vec{0} = (-1.72)3\vec{u} + (-1.72)4\vec{v} + 1.72\vec{w}$

$\vec{x} = (2 - 1.72 \times 3)\vec{u} + (3 - 1.72 \times 4)\vec{v} + 2.72\vec{w}$

nothing special about -1.72

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

→ span is a 3-dimensional subspace of $\mathbb{R}^4$

→ they are a basis for that subspace

① is this a basis for $\mathbb{R}^4$? no: only 3 vectors

② dimension of their span? 3

③ are they linearly independent? yes

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ -6 \\ 0 \\ 14 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 3 \\ 7 \end{bmatrix} \right\}$$

1. is this a basis for $\mathbb{R}^4$? no
2. dimension of their span? 3
3. are they linearly independent? no

orthogonal complement: see 4.3