EECS 245, Winter 2026

LEC 11

Matrices!

→ Read Ch. 5.1, will finish 5.2 soon

My freshman year transcript

Fall 2016

Class	Title	Un.	Gr.
CHEM 1A	General Chemistry	3	B-
CHEM 1AL	General Chemistry Laboratory	1	C+
COMPSCI 61A	The Structure and Interpretation of Computer Programs	4	B+
COMPSCI 70	Discrete Mathematics and Probability Theory	4	A
COMPSCI 195	Social Implications of Computer Technology	1	P
MATH 1A	Calculus	4	A+

Kind of like EECS 245

Spring 2017

Class	Title	Un.	Gr.
COMPSCI 61B	Data Structures	4	B+
COMPSCI 97	Field Study	1	P
COMPSCI 197	Field Study	1	P
ELENG 16A	Designing Information Devices and Systems I	4	B-
MATH 110	Linear Algebra	4	C
MATH 128A	Numerical Analysis	4	B+

Math 217

That said, grades still do matter.

The easiest path to a good grade is to do the labs and homeworks yourself without ChatGPT AND fully understand the solutions

exams 70%, labs + homeworks 30%

On the bright side: don't forget the redemption policy!

Agenda

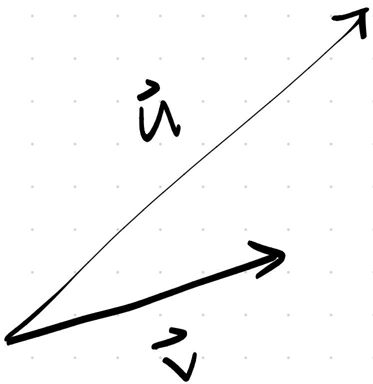
Read 5.1 + 5.2
(5.2 in progress)

- What's the point?
- Matrices
  - Addition/scalar mult.
  - Matrix-vector mult.
  - Matrix-matrix mult.
  - Transpose & identity matrices

Announcements

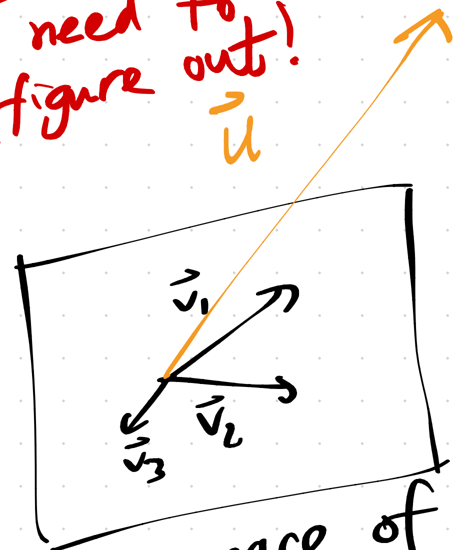
- Midterm regrades due on Tuesday; check solutions AND Grade Report + common misconceptions
- 1-on-1 check-ins with me available
- No lab this week
- HW 6 will come out on Friday

What's the point?



Approximating one vector using one other vector

this is what we need to figure out!



subspace of $\mathbb{R}^n$ (spanned by $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_d$)

Which vector in subspace is closest to $\vec{u}$?

Matrix: rectangular array of numbers

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 1 & 9 \\ 0 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix}_{4\times 3}$$

$\vec{v}_1 \quad \vec{v}_2 \quad \vec{v}_3$

4 rows, 3 columns

4×3

$A \in \mathbb{R}^{4\times 3}$

$A_{23} = 9$

A_{ij} = row i, col j

column vectors, written next to each other

In general, A is $n \times d$

rows

columns

$n > d$

tall

$n = d$

$n < d$

wide

Addition and scalar multiplication

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 1 & 9 \\ 0 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix}_{4\times 3} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}_{4\times 3}$$

we can add A and B because dimensions are the same!

$$3A - B = \begin{bmatrix} & & 9_{1,3} \\ & & \\ & -11_{3,2} & \\ & & \end{bmatrix}_{4\times 3}$$

"Golden Rule" for Matrix Multiplication

In order to be able to multiply

$A_{n \times d}$ and $B_{d \times p}$

inner dimensions

columns in A = # rows in B

AB has shape $n \times p$
(outer dimensions)

Matrix-vector multiplication

$\vec{x} \in \mathbb{R}^3$

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 1 & 9 \\ 0 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

dot product of $\vec{x}$ with every $\underline{\underline{\text{row}}}$ of A

$$\underset{4\times 3}{A}\ \underset{3\times 1}{\vec{x}} = \underset{4\times 1}{\begin{bmatrix} 15 \\ 29 \\ 0 \\ 2 \end{bmatrix}}$$

$$\begin{bmatrix} 2 \\ 1 \\ 9 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 1 & 9 \\ 0 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

$$A\vec{x} = 1\begin{bmatrix} 3 \\ 2 \\ 0 \\ 2 \end{bmatrix} + 0\begin{bmatrix} 1 \\ 1 \\ -1 \\ -2 \end{bmatrix} + 3\begin{bmatrix} 4 \\ 9 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 15 \\ 29 \\ 0 \\ 2 \end{bmatrix}$$

Important

$A\vec{x}$ is a linear combination of the columns in A, with the coefficients from $\vec{x}$

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 1 & 9 \\ 0 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix} \quad 4 \times 3$$

$$B = \begin{bmatrix} 1 & 2 \\ 0 & 7 \\ 3 & 2 \end{bmatrix} \quad 3 \times 2$$

match!

$$AB = \begin{bmatrix} 15 & 21 \\ 29 & 29 \\ 0 & \vdots \\ 2 & \vdots \end{bmatrix} \quad 4 \times 2$$

col j of AB is A · (col j of B)

Properties

- $(AB)C = A(BC)$ associative ✓
- $(A+B)C = AC + BC$ distributive ✓
- NOT COMMUTATIVE!!!

in general, $AB \neq BA$

$A_{n\times d}$ $B_{d\times p}$

AB ✓

$B_{d\times p} A_{n\times d}$

just because AB is a valid product, BA might not be!

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 0 \end{bmatrix}}_{A} 3\times 2 \qquad \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}}_{B} 2\times 3$$

$$\underset{3\times 2 \quad 2\times 3}{AB} \rightarrow 3 \times 3$$

$$BA \rightarrow 2 \times 2$$

even if both are valid, they don't need to have the same shape!

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

even if A, B square, $AB \neq BA$ in general!

$$AB = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix} \qquad BA = \begin{bmatrix} 23 & \\ & \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Activity 4.1 in 5.1

"diagonal matrix"

$$P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad S = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} 4 \\ 6 \\ 12 \end{bmatrix}$$

$$P\vec{x} = \begin{bmatrix} 12 \\ 4 \\ 6 \end{bmatrix}, \quad S\vec{x} = \begin{bmatrix} 16 \\ 3 \\ 36 \end{bmatrix}, \quad PS\vec{x} = \begin{bmatrix} 36 \\ 16 \\ 3 \end{bmatrix}, \quad SP\vec{x} = \begin{bmatrix} 48 \\ 2 \\ 18 \end{bmatrix}$$

$$PS \neq SP$$

identity matrix, I

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{I_2} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 16 & 3 \\ -3 & 4 & 2 \\ \pi & 2 & 0 \end{bmatrix}}_{I_3} = \begin{bmatrix} 2 & 16 & 3 \\ -3 & 4 & 2 \\ \pi & 2 & 0 \end{bmatrix}$$

transpose of a matrix

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \\ 4 & -6 \end{bmatrix}$$

A^T's columns are A's rows

$$(A^T)_{ij} = A_{ji}$$

$$A^T = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 0 & -6 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

Important! Read 5.2

Ponder: Simplify $\|A\vec{x}\|^2$