

EECS 245 Fall 2025 Math for ML

Lecture 11: Matrices

7 Read: Ch. 2.7

About exam grades...

my freshman transcript

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|-------------|---|-----|-----|---|
| Fall 2016 | J | • | | |
| Class | Title | Un. | Gr. | |
| CHEM 1A | General Chemistry | 3 | B- | |
| CHEM 1AL | General Chemistry Laboratory | 1 | C+ | |
| COMPSCI 61A | The Structure and Interpretation of Computer Programs | 4 | B+ | |
| COMPSCI 70 | Discrete Mathematics and Probability Theory | 4 | Α | |
| COMPSCI 195 | Social Implications of Computer Technology | 1 | P | |
| MATH 1A | Calculus | 4 | A+ | |
| Spring 2017 | | | | |
| Class | Title | Un. | Gr. | |
| COMPSCI 61B | Data Structures | 4 | B+ | |
| COMPSCI 97 | Field Study | 1 | P | |
| COMPSCI 197 | Field Study | 1 | Р | |

Designing Information Devices and Systems I

Linear Algebra

Numerical Analysis

th 217

ELENG 16A

MATH 128A

MATH 110

but still, grades do matter, and the easiest path to a good grade is to actually do the labs and homeworks yourself WITHOUT ChatGPT otherwise, you're just cheating yourself!

exams 70%, homeworks 30%

Agenda

Matrices: definition, addition, scalar multiplication

2 Matrix-vector multiplication

3 Matrix-matrix multiplication

(If time, otherwise Tuesday) rank, CR de composition

HW 5 out tomorrow!

Matrix: Rectangular grid of numbers

In general,

A
$$\in \mathbb{R}^{n \times d}$$
 for columns

$$\begin{bmatrix}
1 & 1 \\
3 & 4 \\
-2 & 0 \\
5 & 0
\end{bmatrix}
\begin{bmatrix}
2 & 3 \\
-1 & 2
\end{bmatrix}
\begin{bmatrix}
2 & 0 & 3 & 1 & 0 \\
1 & 0 & 5 & 1 & 5 \\
3 & 0 & 8 & 1 & 3
\end{bmatrix}$$
"square"

"wide"

 $n > d$
 $n = d$
 $n < d$

matrices support addition and scalar multiplication

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

"Golden rule" of matrix multiplication

Suppose A, B matrices.

In order for the product

to exist,

columns in A = # rows in B

Matrix-vector

think of a vector is just a matrix with I column

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix} \quad \dot{\vec{x}} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

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the dot product of
$$\vec{x}$$
 with every row of A ?

 $4 \times 3 \times 3 \times 1 = 4 \times 1$

output

$$A = \begin{bmatrix} 3 \\ 2 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

$$\frac{-2}{x} = \frac{3}{(3)(1) + (1)(3) + (4)(3)}$$

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"Computational"
interpretation

conceptual

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 0 \\ 3 & 1 & 0 \\ 2 & -2 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix} + 0 \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 0 \end{bmatrix} + 3 \begin{bmatrix} 4 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

linear combination of the columns in A, using the coefficients in X

M= [2 -1 3 0 4]

1 5 -2 1 0

each description is of a new vector
$$\vec{u}$$
:

1) a vector whose second component is (, rest are 0)

2) a vector with all components = 1/5

3) a vector with first component = 3/5 \vec{u} = 1/10

weighted mean of columns

$$M = \begin{bmatrix} 2 & -1 & 3 & 0 & 4 \\ 1 & 5 & -2 & 1 & 0 \end{bmatrix}$$

$$u = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 \end{bmatrix}$$

$$Mu = \begin{bmatrix} -1 & 1 & 1 \\ 5 & 1 & 1 \end{bmatrix}$$

| $U = \frac{1}{15}$ | $U = \frac{2(\frac{1}{5}) + (-1)(\frac{1}{5})}{15}$ | $U = \frac{1}{15}$ | $U = \frac{1}{15$

Matrix -matrix multiplication

In general, BERDXP A ER Xd AB ER R = (row i in A) · (co lumn j in B)

properties i) (AB) C = A(BC)
as long as shapes satisfy "Golden
Rule" A(B+C) = AB+AC AB≠BA e up of different dot products

Asx3 B3 x 6

AB exists,

but BAA doesn't

3x6 (sx3)

don't metch?

A 3x6 B 6 x3

both exist, but have different dimensions

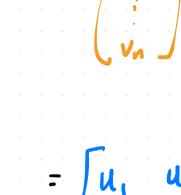
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

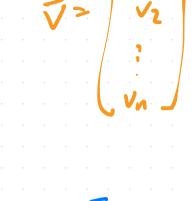
$$B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$
even if AB, BA
same shape
oven't equal in general!
$$BA = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$BA = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

 $BA = \begin{bmatrix} 23 & 34 \\ 71 & 46 \end{bmatrix}$ row 2 column 2 of B of A

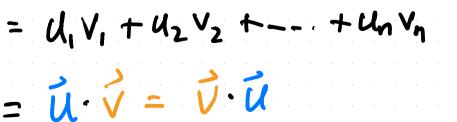
matrix Transpose of rows -> columns 2 wor columns -







$$= \begin{bmatrix} u_1 & u_2 & \dots & u_n \end{bmatrix}$$



important:

$$(AB)^{T} = B^{T}A^{T}$$

application: $\|A\vec{x}\|^2 = (A\vec{x}) \cdot (A\vec{x}) = (A\vec{x})^T (A\vec{x})$ vector $= \vec{x}^T A^T A \vec{x}$