Lab 5: Vector Spaces, Subspaces, Bases, and Dimension

EECS 245, Spring 2026 at the University of Michigan

due for completion at 11:59PM Ann Arbor Time on Wednesday, May 20th, 2026

Name: ______________________________

uniqname: ______________________________

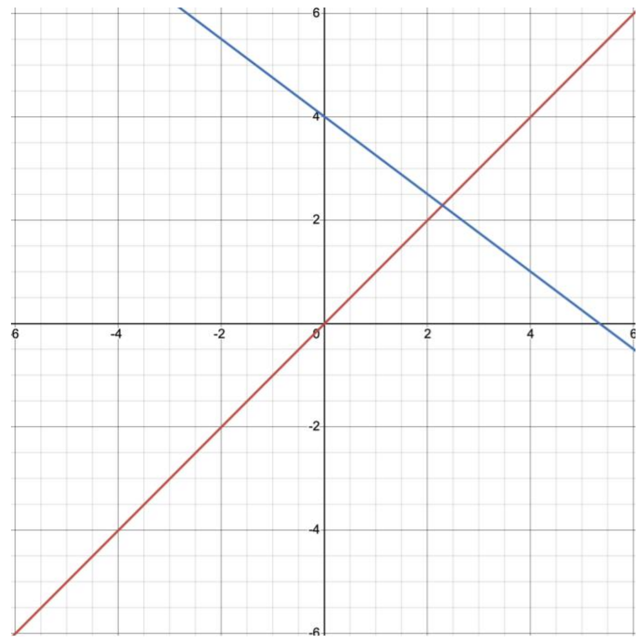
Each lab worksheet will contain several activities, some of which will involve writing code and others that will involve writing math on paper. To receive credit for a lab, you must complete as many of the activities as you can in 2 hours and submit a PDF of your work to Gradescope. We will provide specific instructions on how to submit programming activities (e.g. submitting the notebook or including a screenshot of some output).

Feel free to work with others in the course, but you must submit individually.

Recap: Vector Spaces, Subspaces, Bases, and Dimension (**Chapter 4.3**)

- A **subspace** S of a vector space V is a set of vectors where:
  1. $\vec{0} \in S$
  2. $\vec{u}, \vec{v} \in S \rightarrow \vec{u} + \vec{v} \in S$
  3. $\vec{u} \in S, c \in \mathbb{R} \rightarrow c\vec{u} \in S$

  If you take any two vectors $\vec{u}, \vec{v} \in S$, then any linear combination $c\vec{u} + d\vec{v}$ must also be in S.
- As an example, let's consider $\mathbb{R}^2$, which itself is a vector space.



- The line through the origin **is** a subspace of $\mathbb{R}^2$, with dimension 1. It is the span of the vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.
- The other line, however, is **not** a subspace of $\mathbb{R}^2$, since it doesn't pass through the origin.

- A **basis** for a subspace S is a set of vectors that:
  1. span all of S
  2. are linearly independent

  A basis for a subspace is a minimal set of vectors that spans the whole subspace. All subspaces have infinitely many bases. For example, $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ and $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$ are both bases for $\mathbb{R}^2$.
- The **dimension** of a subspace S, denoted $\dim(S)$, is the number of vectors in any basis for S.

Activity 1: Formal Definition of Linear Independence

Suppose $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_d \in \mathbb{R}^n$, and that $\vec{b} \in \text{span}(\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_d\})$.

a) Give a one sentence English explanation of what it means for $\vec{b} \in \text{span}(\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_d\})$.

b) Suppose that $a_1\vec{v}_1 + a_2\vec{v}_2 + \ldots + a_d\vec{v}_d = \vec{b}$ **and** $c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_d\vec{v}_d = \vec{b}$, where at least one of the a_i's is different from its corresponding c_i.
Using the formal definition of linear independence from Chapter 4.2, determine whether or not $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_d$ are linearly independent, and prove your answer.

c) Find another set of coefficients $k_1, k_2, \ldots, k_d$ such that

$$k_1\vec{v}_1 + k_2\vec{v}_2 + \ldots + k_d\vec{v}_d = \vec{b}$$

and at least one of the k_i's is different from its corresponding a_i or c_i.

By doing this, you're showing that if there is at least one way to write $\vec{b}$ as a linear combination of a set of vectors, then there are infinitely many ways to write $\vec{b}$ as a linear combination of those vectors; there can't just be two or three ways to do it.

Activity 2: Thinking in Higher Dimensions

a) Suppose $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_8$ are 8 vectors in $\mathbb{R}^5$. Fill in each blank below with one of the provided options, and explain your reasoning.

(i) These vectors ________ span all of $\mathbb{R}^5$.
(options: do, do not, might)

(ii) These vectors ________ linearly independent.
(options: are, are not, might be)

(iii) Any 5 of these vectors ________ span all of $\mathbb{R}^5$.
(options: do, do not, might)

b) Suppose $\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_{10}$ are 10 non-zero vectors in $\mathbb{R}^{11}$.

Furthermore, suppose that $\text{span}(\{\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_{10}\})$ is a 6-dimensional subspace of $\mathbb{R}^{11}$. This means that there exists a subset of 6 of these vectors that is linearly independent and spans the same 6-dimensional subspace as the original 10 vectors; we just don't know which 6.

(i) Let k be the dimension of the subspace spanned by a subset of 4 of these vectors. What are all possible values of k?

(ii) Let m be the dimension of the subspace spanned by a subset of 7 of these vectors. What are all possible values of m?

Activity 3: Introduction to Subspaces

Only one of the following is a subspace of $\mathbb{R}^3$. Which one? Explain why the others are not subspaces.

The set of vectors $\vec{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\mathbb{R}^3$ such that

(i) $x + 2y - 3z = 4$

(ii) $\vec{v}$ is on the line $L = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, t \in \mathbb{R}$

(iii) $x + y + z = 0$ and $x - y + z = 1$

(iv) $x = -z$ and $x = z$

(v) $x^2 + y^2 = z$

Activity 4: Finding Non-Examples of Subspaces

In this activity, you'll find sets of vectors in $\mathbb{R}^2$ that satisfy some, but not all, of the requirements for a subspace. Think creatively, and since we're working in $\mathbb{R}^2$, visualize the vectors!

a) Find a set of vectors in $\mathbb{R}^2$ such that the sum of any two vectors $\vec{u}$ and $\vec{v}$ in the set is also in the set, but $\frac{1}{2}\vec{v}$ is possibly not in the set.

b) Find a set of vectors in $\mathbb{R}^2$ such that $c\vec{v}$ is in the set for any vector $\vec{v}$ in the set and any scalar c, but the sum of any two vectors $\vec{u}$ and $\vec{v}$ in the set is possibly not in the set.

Activity 5: Finding Bases for Subspaces

In each part below, find **two different possible bases** for the given subspace, and state the **dimension** of the subspace.

a) $S = \text{span}\left(\left\{\begin{bmatrix}1\\3\\3\end{bmatrix}, \begin{bmatrix}-3\\-9\\-9\end{bmatrix}, \begin{bmatrix}1\\5\\-1\end{bmatrix}, \begin{bmatrix}2\\7\\4\end{bmatrix}, \begin{bmatrix}1\\4\\1\end{bmatrix}\right\}\right)$

b) $S = \left\{\begin{bmatrix}v_1\\v_2\end{bmatrix} \mid v_1 = -v_2; v_1, v_2 \in \mathbb{R}\right\}$

c) $S = \left\{\begin{bmatrix}v_1\\v_2\\v_3\\v_4\end{bmatrix} \mid v_4 = 0; v_1, v_2, v_3 \in \mathbb{R}\right\}$