Lab 3: Simple Linear Regression and Partial Derivatives

EECS 245, Winter 2026 at the University of Michigan

due by the end of your lab section

Name: ____________________

uniqname: ____________________

Each lab worksheet will contain several activities, some of which will involve writing code and others that will involve writing math on paper. To receive credit for a lab, you must complete all activities and show your lab TA by the end of the lab section.

While you must get checked off by your lab TA **individually**, we encourage you to form groups with 1-2 other students to complete the activities together.

Recap: Simple Linear Regression

We've spent all of Chapter 2 learning about the simple linear regression model, $h(x_i) = w_0 + w_1 x_i$.

To find the optimal intercept, w_0^*, and slope, w_1^*, we minimized mean squared error:

$$R_{\text{sq}}(w_0, w_1) = \frac{1}{n}\sum_{i=1}^{n}(y_i - (w_0 + w_1 x_i))^2$$

- $R_{\textbf{sq}}$ **is a function of** w_0 **and** w_1**, and looks like a bowl in 3D.** Since it has two input variables, we found its minimum by taking the partial derivatives of $R_{\text{sq}}(w_0, w_1)$ with respect to w_0 and w_1, setting both of them equal to 0, and then solving for the resulting w_0^* and w_1^*.
- A partial derivative is defined as the derivative with respect to one variable **while treating all others as constants**.

$$f(x, y) = x^2 + 3xy^2 \implies \frac{\partial f}{\partial x} = 2x + 3y^2$$

- An important fact about the line $h^*(x_i) = w_0^* + w_1^* x_i$ is that it is guaranteed to pass through $(\bar{x}, \bar{y})$ — in other words, an average input always predicts an average output.
- There are several equivalent ways to write the optimal slope, w_1^*. One of them involves the correlation coefficient, r.

$$\underbrace{r = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{\sigma_x}\right)\left(\frac{y_i - \bar{y}}{\sigma_y}\right)}_{\text{average product of } x \text{ and } y\text{, once both are standardized}} \qquad w_1^* = r\frac{\sigma_y}{\sigma_x} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}, \quad w_0^* = \bar{y} - w_1^*\bar{x}$$

Activity 1: The Meaning of Mean Squared Error

Suppose we'd like to predict the number of minutes a delivery will take, y, as a function of distance, x. To do so, we look to our dataset of n deliveries, $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$, and fit two simple linear models:

- $F(x_i) = a_0 + a_1 x_i$, where:
$$a_1 = r\frac{\sigma_y}{\sigma_x}, \qquad a_0 = \bar{y} - a_1\bar{x}$$
Here, r is the correlation coefficient between x and y, $\bar{x}$ and $\bar{y}$ are their respective means, and σ_x and σ_y are their respective standard deviations.
- $G(x_i) = b_0 + b_1 x_i$, where b_0 and b_1 are chosen such that $G(x_i) = b_0 + b_1 x_i$ minimizes **mean absolute error** on the dataset. Assume that no other line minimizes mean absolute error on the dataset, i.e. that the values of b_0 and b_1 are unique.

a) Fill in the $\boxed{???}$:

$$\sum_{i=1}^{n}(y_i - F(x_i))^2 \quad \boxed{???} \quad \sum_{i=1}^{n}(y_i - G(x_i))^2$$

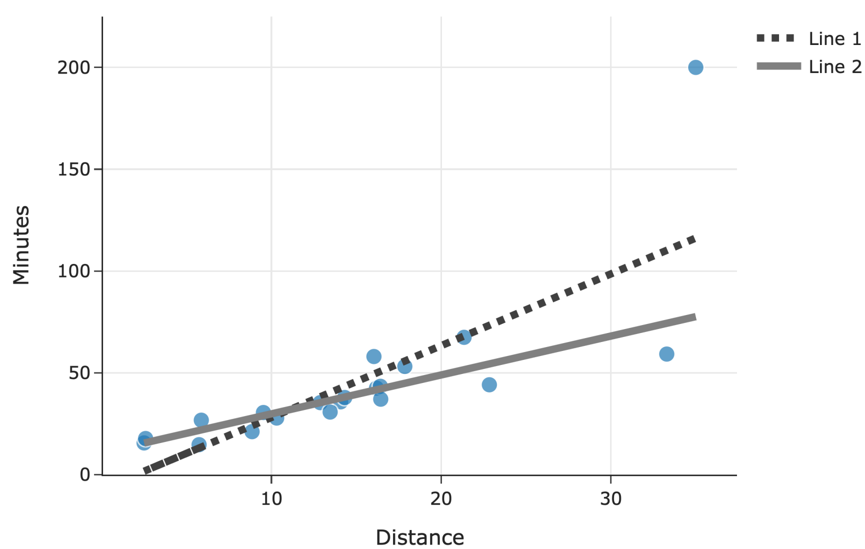
○ $>$ ○ $\geq$ ○ $=$ ○ $\leq$ ○ $<$ ○ Impossible to tell

b) Fill in the $\boxed{???}$:

$$\left(\sum_{i=1}^{n}|y_i - F(x_i)|\right)^2 \quad \boxed{???} \quad \left(\sum_{i=1}^{n}|y_i - G(x_i)|\right)^2$$

○ $>$ ○ $\geq$ ○ $=$ ○ $\leq$ ○ $<$ ○ Impossible to tell

c) Below, we've drawn the lines for both F and G along with a scatter plot for the original n deliveries:



Which line corresponds to F? ○ Line 1 ○ Line 2

Activity 2: What Do You Mean?

Suppose we want to fit a simple linear model (using squared loss) that predicts the number of ingredients in a product given its price. We're given that:

- The average cost of a product in our dataset is $40, i.e. $\bar{x} = 40$
- The average number of ingredients in a product in our dataset is 15, i.e. $\bar{y} = 15$

The intercept and slope of the regression line are $w_0^* = 11$ and $w_1^* = \frac{1}{10}$, respectively.

a) Suppose Victors' Veil (a skincare product) costs $40 and has 11 ingredients. What is the squared loss of our model's predicted number of ingredients for Victors' Veil?

b) Is it possible to answer part **a)** above **just** by knowing $\bar{x}$ and $\bar{y}$, i.e. **without** knowing the values of w_0^* and w_1^*? Once you select an answer, explain it to your peers.

◯ Yes, it's possible ◯ No, it's not possible

Activity 3: Reverse Regression

Suppose we have a dataset of n houses that were recently sold in the Ann Arbor area. For each house, we have its square footage and most recent sale price. The correlation between square footage and price is r.

First, we minimize mean squared error to fit a simple linear model that uses square footage to predict price. The resulting regression line has an intercept of w_0^* and slope of w_1^*.

$$\text{predicted price}_i = w_0^* + w_1^* \cdot \text{square footage}_i$$

We're now interested in minimizing mean squared error to fit a simple linear model **that uses price to predict square footage** — that is, we're "reversing" the x and y variables. Suppose this new regression line has an intercept of β_0^* and slope of β_1^*.

Find β_1^*. Give your answer in terms of one or more of n, r, w_0^*, and w_1^*.

Activity 4: Partial Derivatives and Minimization

Consider the function

$$g(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

a) Find $\frac{\partial g}{\partial x_1}$ and $\frac{\partial g}{\partial x_2}$, the partial derivatives of g with respect to x_1 and x_2.

b) Find the values of x_1 and x_2 that minimize g. You do not need to use the second derivative test to verify that you've found a minimum. (In fact, "the second derivative test" for functions with multiple input variables is much more complicated, and involves linear algebra.)

Activity 5: Systems of Equations

Next week, we'll start learning about vectors, and various applications of them will involve solving systems of equations. Here, you'll practice solving systems of equations with three variables.

In each of the following systems of equations, solve for x_1, x_2, and x_3. If you cannot find a unique solution, explain why.

a)

$$\begin{aligned} -4x_1 + 7x_2 - 2x_3 &= 2 \\ x_1 - 2x_2 + x_3 &= 3 \\ 2x_1 - 3x_2 + x_3 &= -4 \end{aligned}$$

b)

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 4 \\ 2x_1 + 4x_2 - 2x_3 &= 8 \\ x_1 - x_2 + 3x_3 &= 1 \end{aligned}$$

The rest of this worksheet is extra practice (taken from past exams that Suraj wrote). Don't feel pressured to answer all of these problems in lab, but make sure to attempt them at some point.

Activity 6: Transformed Data

Suppose we're given a dataset of n points, $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, where $\bar{x}$ is the mean of $x_1, x_2, \ldots, x_n$ and $\bar{y}$ is the mean of $y_1, y_2, \ldots, y_n$.

Using this dataset, we create a *transformed* dataset of n points, $(x'_1, y'_1), (x'_2, y'_2), \ldots, (x'_n, y'_n)$, where:

$$x'_i = 4x_i - 3 \qquad y'_i = y_i + 24$$

So the transformed dataset is of the form

$$(4x_1 - 3, y_1 + 24), (4x_2 - 3, y_2 + 24), \ldots, (4x_n - 3, y_n + 24)$$

We decide to fit a simple linear model $h(x'_i) = w_0 + w_1 x'_i$ on the transformed dataset using squared loss. We find that $w_0^* = 7$ and $w_1^* = 2$, so $h^*(x'_i) = 7 + 2x'_i$.

a) Suppose we were to fit a simple linear model through the original dataset, $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, again using squared loss. What would the optimal slope on the original dataset be?

◯ 2 ◯ 4 ◯ 6 ◯ 8 ◯ 11 ◯ 12 ◯ 24

b) Recall, the model $h^*(x'_i) = w_0 + w_1 x'_i$ was fit on the transformed dataset, $(x'_1, y'_1), (x'_2, y'_2), \ldots, (x'_n, y'_n)$. $h^*(x'_i)$ happens to pass through the point $(\bar{x}, \bar{y})$. What is the value of $\bar{x}$? Give your answer as an integer with no variables. *Hint: What else does $h^*(x'_i)$ pass through?*

Activity 7: A Refresher

Consider a dataset of $y_1, y_2, \ldots, y_n$, all of which are **positive**. We want to fit a constant model, $h(x_i) = w$, to the data.

Let w_p^* be the optimal constant prediction that minimizes average degree-p loss, $R_p(w)$, defined below:

$$R_p(w) = \frac{1}{n} \sum_{i=1}^{n} |y_i - w|^p$$

For example, w_2^* is the optimal constant prediction that minimizes $R_2(w) = \frac{1}{n} \sum_{i=1}^{n} |y_i - w|^2$

a) In each of the parts below, determine the value of the quantity provided. By "the data", we are referring to $y_1, y_2, \ldots, y_n$. The answer choices are as follows; **select one item in each row**.

A: The standard deviation of the data

B: The variance of the data

C: The mean of the data

D: The median of the data

E: The midrange of the data, $\frac{y_{\text{min}} + y_{\text{max}}}{2}$

F: The mode of the data

G: None of these

		A	B	C	D	E	F	G
(i)	h_0^*	○	○	○	○	○	○	○
(ii)	h_1^*	○	○	○	○	○	○	○
(iii)	$R_1(h_1^*)$	○	○	○	○	○	○	○
(iv)	h_2^*	○	○	○	○	○	○	○
(v)	$R_2(h_2^*)$	○	○	○	○	○	○	○

b) Now, suppose we want to find the optimal constant prediction, h_{U}^*, using the "Ulta" loss function, defined below:

$$L_{\text{U}}(y_i, w) = y_i(y_i - w)^2$$

To find h_{U}^*, we minimize $R_{\text{U}}(w)$, the average Ulta loss. How does h_{U}^* compare to the mean of the data, M?

○ $h_{\text{U}}^* > M$ ○ $h_{\text{U}}^* \geq M$ ○ $h_{\text{U}}^* = M$ ○ $h_{\text{U}}^* \leq M$ ○ $h_{\text{U}}^* < M$

c) Finally, to find the optimal constant prediction, we will instead minimize **regularized** average Ulta loss, $R_\lambda(w)$, defined below:

$$R_\lambda(w) = \left(\frac{1}{n} \sum_{i=1}^{n} y_i (y_i - w)^2 \right) + \lambda w^2$$

Here, assume $\lambda > 0$ is some positive constant. (We will cover regularization in more detail later in the term.)

Find w^*, the constant prediction that minimizes $R_\lambda(w)$. Give your answer as an expression in terms of the y_i's, n, and/or λ.